ONLINE APPENDIX TO "VOLUNTARY DISCLOSURE IN ASYMMETRIC CONTESTS" BY CHRISTIAN EWERHART AND JULIA LAREIDA, OCTOBER 31, 2023

This Appendix contains technical proofs and other material that has been omitted from the body of the paper. The content of the Appendix is organized as follows.

A. Material omitted from Section 2

Below, we outline the formal details regarding the Bayesian updating as well as the proof of Lemma 1.

A.1 Bayesian updating
Fix a contestant $i \in \{1, 2\}$, and suppose given a set of revealing types, $S_i \subseteq C_i$. Then, there are three scenarios: (i) Suppose first that player i discloses $c_i \in C_i$. Then, player i is believed to be of type c_i with probability one, i.e., $\mu_i(c_i) = 1$. (ii) Next, suppose that player i does not disclose her type, and that player i's decision to not disclose is a possibility on the equilibrium path, i.e., $S_i \subsetneq C_i$. Then, c_i is expected to be in the set-theoretic complement of S_i. Hence, by Bayes' rule, $\mu_i(c_i) = q_i(c_i)/\sum_{c_i' \in C_i \backslash S_i} q_i(c_i')$ if $c_i \in C_i \backslash S_i$, while $\mu_i(c_i) = 0$ if $c_i \in S_i$. (iii) Finally, suppose that player i does not disclose her type, and that i's decision to not disclose is an off-equilibrium event, i.e., $S_i = C_i$. Then, the belief about player i may be specified by any $\mu_i = \mu_i^0 \in \Delta(C_i)$.

A.2 Proof of Lemma 1
Lemma 1 concerns the existence and uniqueness of the Bayesian equilibrium at the contest stage.

Proof of Lemma 1. This is a special case of a result in Ewerhart and Quartieri (2020). □

Lemma 1 extends to randomized bids. Indeed, since each player is active with positive probability, and payoffs functions are l.s.c. in the own bid at the origin, expected payoffs against the opponent's equilibrium strategy are strictly concave over $\mathbb{R}_+$, so that it is suboptimal to randomize strictly.

B. Material omitted from Section 3

This section presents three auxiliary results and the proof of Lemma 2.

B.1 Wärneryd's transformation

The function introduced in the following lemma arises naturally in the first-order conditions.[1]

Lemma B.1 (Wärneryd's transformation) *Let $\Phi(z) = h(z)/h'(z)$, for $z > 0$. Then, the following holds true: (i) $\lim_{z \searrow 0} \Phi(z) = 0$; (ii) $1 \leq \Phi' \leq \underline{\rho}$; (iii) $(d \ln h)/(d \ln \Phi) = 1/\Phi'$; (iv) if $x_i > 0$, then player i's best-response mapping in the complete-information contest is differentiable with*

$$\frac{dx_i}{dx_j} = \frac{\Phi(x_i)}{\Phi(x_j)} \frac{2p_i - 1}{\Phi'(x_i) - 1 + 2p_i}, \tag{B.1}$$

where $i, j \in \{1, 2\}$ with $j \neq i$, and $p_i = p_i(x_i, x_j)$.

Proof. (i) By assumption, h is differentiable in the interior of the strategy space, with h' positive and declining. Hence, $\lim_{z \to 0} h'(z) \in (0, \infty]$. Moreover, by continuity, $\lim_{z \to 0} h(z) = 0$. The claim follows. (ii) Note first that $\Phi' = 1 - (hh''/(h')^2) \geq 1$ by the concavity of h. To see that $\Phi' \leq \underline{\rho}$, take some $\rho > \underline{\rho}$ such that h^ρ is convex. Then, in the interior of the strategy space, $\rho(\rho - 1)h^{\rho-2}(h')^2 + \rho h^{\rho-1}h'' \geq 0$. Recall that $\underline{\rho} \geq 1$. Hence, $\rho > 1$. Dividing by $\rho h^{\rho-2}(h')^2 > 0$, and rearranging, one obtains $\Phi' \leq \rho$. Taking the limit $\rho \to \underline{\rho}$, the claim follows. (iii) A straightforward calculation shows that

$$\frac{d \ln h(z)}{d \ln \Phi(z)} = \left(\frac{dh(z)}{h(z)}\right) \bigg/ \left(\frac{d\Phi(z)}{\Phi(z)}\right) = \frac{h'(z)dz}{h(z)} \cdot \frac{\Phi(z)}{\Phi'(z)dz} = \frac{1}{\Phi'(z)} \qquad (z > 0), \tag{B.2}$$

as claimed. (iv) The first-order condition characterizing the best response x_i reads $p_i(1 - p_i) = c_i \Phi(x_i)$. Total differentiation delivers $(1 - 2p_i)dp_i = c_i \Phi'(x_i)dx_i$, where

$$dp_i = \frac{p_i(1 - p_i)}{\Phi(x_i)} dx_i - \frac{p_i(1 - p_i)}{\Phi(x_j)} dx_j = c_i dx_i - c_i \frac{\Phi(x_i)}{\Phi(x_j)} dx_j. \tag{B.3}$$

Simplifying, we obtain (B.1). □

B.2 Monotonicity of best-response bid schedules

Best-response bid schedules are monotone declining in marginal cost, and strictly so in the interior.

Lemma B.2 (Monotonicity of best-response bid schedules) *Let $\xi_j \in X_j^*$ and $c_i, \widehat{c}_i \in C_i$ for $i \neq j$ such that $c_i > \widehat{c}_i$. Then, $\widetilde{\beta}_i(\xi_j; c_i) \leq \widetilde{\beta}_i(\xi_j; \widehat{c}_i)$, where the inequality is strict if $\widetilde{\beta}_i(\xi_j; \widehat{c}_i) > 0$.*

Proof. Take a bid schedule $\xi_j \in X_j^*$. The assertion is obvious for $\widetilde{\beta}_i(\xi_j; c_i) = 0$. Suppose instead that $x_i \equiv \widetilde{\beta}_i(\xi_j; c_i) > 0$. Then, from c_i's first-order condition, $\partial E_{c_j}[p_i(x_i, \xi_j(c_j))]/\partial x_i = c_i$. We will show first that the left-hand side of this equation is strictly declining in x_i. Indeed, because the best-response bid $\widetilde{\beta}_i(\xi_j; c_i)$ exists, there is some $c_j \in C_j$ such that $\xi_j(c_j) > 0$. A straightforward calculation shows, therefore, that

$$\frac{\partial^2 E_{c_j}[p_i(x_i, \xi_j(c_j))]}{\partial x_i^2} = \frac{\partial}{\partial x_i} E_{c_j} \left[\frac{\gamma_i \gamma_j h'(x_i) h(\xi_j(c_j))}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^2}\right] \tag{B.4}$$

$$= E_{c_j} \left[\frac{\gamma_i \gamma_j h(\xi_j(c_j)) \left\{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))h''(x_i) - 2\gamma_i (h'(x_i))^2\right\}}{(\gamma_i h(x_i) + \gamma_j h(\xi_j(c_j)))^3}\right] < 0, \tag{B.5}$$

[1] Cf. Wärneryd (2003) and Inderst et al. (2007).

which proves the claim. There are now two cases. Assume first that $\widehat{x}_i > 0$. For this case, it is claimed that $\widehat{x}_i > x_i$. To provoke a contradiction, suppose that $\widehat{x}_i \leq x_i$. Then, since the marginal probability of winning for player i is strictly declining in i's bid, $\widehat{c}_i = \partial E_{c_j}[p_i(\widehat{x}_i, \xi_j(c_j))]/\partial x_i \geq \partial E_{c_j}[p_i(x_i, \xi_j(c_j))]/\partial x_i = c_i$, in conflict with $\widehat{c}_i < c_i$. Hence, $\widehat{x}_i > x_i$, as claimed. Assume next that $\widehat{x}_i = 0$, i.e., type $\widehat{c}_i$ finds it optimal to respond to ξ_j with a zero effort. But then, strictly higher marginal costs induce type c_i to do the same, i.e., $x_i = 0$. The lemma follows. □

B.3 Bounds on the bid distributions

From the first-order conditions, we derive upper and lower bounds on active contestants' bid distributions.

Lemma B.3 (Bounds on the bid distributions) *Let $\xi^* = (\xi_1^*, \xi_2^*)$ be a Bayesian equilibrium in an incomplete-information contest such that both players are active with probability one. Then,*

$$\gamma_i h(\xi_i^*(\underline{c}_i)) \leq \frac{1}{\pi_i} \cdot \gamma_i h(\xi_i^*(\overline{c}_i)) + \frac{1-\pi_i}{\pi_i} \cdot \gamma_j h(\xi_j^*(\underline{c}_j)) \quad (i, j \in \{1, 2\},\, j \neq i), \tag{B.6}$$

$$h(\xi_2^*(\overline{c}_2)) \leq \frac{1}{\widehat{\sigma}} \cdot h(\xi_1^*(\underline{c}_1)), \tag{B.7}$$

where $\widehat{\sigma} = \sigma$ if $\sigma \leq 1$ and *$\widehat{\sigma} = \sigma^{1/\varrho}$ if $\sigma > 1$.*

Proof. Take an arbitrary type $c_i \in C_i$ of player i. Since, by assumption, $\xi_i^*(c_i) > 0$, the necessary first-order condition for type c_i holds, i.e.,

$$E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(c_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(c_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] - c_i = 0, \tag{B.8}$$

where $j \neq i$. To prove the first claim, evaluate (B.8) at $c_i = \overline{c}_i$. Then, making use of Lemma B.2 and the concavity of h, we get

$$\overline{c}_i = E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2\right] \tag{B.9}$$

$$= E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2} \cdot \underbrace{\left(1 + \frac{\gamma_i h(\xi_i^*(\underline{c}_i)) - \gamma_j h(\xi_i^*(\overline{c}_i))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(c_j))}\right)^2}_{\text{monotone increasing in } c_j}\right] \tag{B.10}$$

$$\geq E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\overline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 \tag{B.11}$$

$$= E_{c_j}\left[\frac{\gamma_i \gamma_j h'(\xi_i^*(\underline{c}_i)) h(\xi_j^*(c_j))}{(\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(c_j)))^2}\right] \times \underbrace{\left(\frac{h'(\xi_i^*(\overline{c}_i))}{h'(\xi_i^*(\underline{c}_i))}\right)}_{\geq 1} \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2 \tag{B.12}$$

$$\geq \underline{c}_i \cdot \left(\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}\right)^2. \tag{B.13}$$

Dividing by $\underline{c}_i > 0$, and using $\pi_i = \sqrt{\underline{c}_i / \overline{c}_i}$, we obtain

$$\frac{\gamma_i h(\xi_i^*(\underline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))}{\gamma_i h(\xi_i^*(\overline{c}_i)) + \gamma_j h(\xi_j^*(\underline{c}_j))} \leq \frac{1}{\pi_i}. \tag{B.14}$$

Inequality (B.6) follows. To prove the second claim, one multiplies type c_i's first-order condition (B.8) by $\Phi(\xi_i^*(c_i))$, and subsequently takes expectations. This yields

$$E_{c_i}[c_i\Phi(\xi_i^*(c_i))] = E_{c_1,c_2}\left[\frac{\gamma_1\gamma_2 h(\xi_1^*(c_1))h(\xi_2^*(c_2))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2}\right] \qquad (i = 1, 2), \tag{B.15}$$

where $E_{c_1,c_2}[.]$ denotes the ex-ante expectation. Exploiting the fact that equilibrium bid schedules are monotone declining (by Lemma B.2), and that $\Phi' > 0$, this implies

$$\underline{c}_2\Phi(\xi_2^*(\overline{c}_2)) \leq E_{c_2}[c_2\Phi(\xi_2^*(c_2))] = E_{c_1}[c_1\Phi(\xi_1^*(c_1))] \leq \overline{c}_1\Phi(\xi_1^*(\underline{c}_1)), \tag{B.16}$$

or, using that $\Phi(\xi_2^*(\overline{c}_2)) > 0$,

$$\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\overline{c}_2))} \geq \frac{\underline{c}_2}{\overline{c}_1} = \sigma. \tag{B.17}$$

There are two cases. Assume first that $\xi_1^*(\underline{c}_1) \geq \xi_2^*(\overline{c}_2)$. Then, using $\Phi' \leq \underline{\rho}$ (see Lemma B.1), we obtain

$$\ln\left(\frac{h(\xi_1^*(\underline{c}_1))}{h(\xi_2^*(\overline{c}_2))}\right) = \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln h(z) \tag{B.18}$$

$$= \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{d\ln h(z)}{d\ln\Phi(z)} d\ln\Phi(z) \tag{B.19}$$

$$= \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} \frac{1}{\Phi'(z)} d\ln\Phi(z) \tag{B.20}$$

$$\geq \frac{1}{\underline{\rho}} \int_{\xi_2^*(\overline{c}_2)}^{\xi_1^*(\underline{c}_1)} d\ln\Phi(z) \tag{B.21}$$

$$= \frac{1}{\underline{\rho}} \ln\left(\frac{\Phi(\xi_1^*(\underline{c}_1))}{\Phi(\xi_2^*(\overline{c}_2))}\right). \tag{B.22}$$

Using (B.17), this implies $h(\xi_2^*(\overline{c}_2)) \leq \sigma^{-1/\underline{\rho}} \cdot h(\xi_1^*(\underline{c}_1))$. Assume next that $\xi_1^*(\underline{c}_1) < \xi_2^*(\overline{c}_2)$. Then using $\Phi' \geq 1$ (taken likewise from Lemma B.1) delivers

$$\ln\left(\frac{h(\xi_2^*(\overline{c}_2))}{h(\xi_1^*(\underline{c}_1))}\right) = \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\overline{c}_2)} \frac{d\ln\Phi(z)}{\Phi'(z)} \leq \int_{\xi_1^*(\underline{c}_1)}^{\xi_2^*(\overline{c}_2)} d\ln\Phi(z) = \ln\left(\frac{\Phi(\xi_2^*(\overline{c}_2))}{\Phi(\xi_1^*(\underline{c}_1))}\right). \tag{B.23}$$

Hence, in that case, $h(\xi_2^*(\overline{c}_2)) \leq \frac{1}{\sigma} \cdot h(\xi_1^*(\underline{c}_1))$. Thus, exploiting that $\underline{\rho} \geq 1$, we conclude that $h(\xi_2^*(\overline{c}_2)) \leq h(\xi_1^*(\underline{c}_1)) \cdot \max\{\sigma^{-1}, \sigma^{-1/\underline{\rho}}\}$. Clearly, this proves (B.7). □

B.4 Proof of Lemma 2

Next, we establish the condition sufficient for uniform asymmetry stated as Lemma 2.

Proof of Lemma 2. Lemma 2 is derived by combining several inequalities, all of which are derived from the first-order conditions necessary for players' bid schedules to be mutual best responses. Property (ii) of Definition 1 will be checked first. Suppose that all types of both players are active. There are two cases.

Case A. Suppose first that $\text{Supp}(\mu_1) = C_1$ *and* $\text{Supp}(\mu_2) = C_2$. We make use of Lemma B.3. Letting $i = 2$ in (B.6) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \frac{1}{\pi_2} \cdot \gamma_2 h(\xi_2^*(\overline{c}_2)) + \frac{1-\pi_2}{\pi_2} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)). \tag{B.24}$$

Combining this with (B.7) delivers

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{ \frac{1}{\pi_2} \cdot \frac{\gamma}{\widehat{\sigma}} + \frac{1-\pi_2}{\pi_2} \right\}}_{\equiv \alpha} \cdot \gamma_1 h(\xi_1^*(\underline{c}_1)), \tag{B.25}$$

where $\gamma = \gamma_2/\gamma_1$, as before. Letting $i = 1$ in (B.6), and plugging the result into (B.25) yields

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \alpha \cdot \left\{ \frac{1}{\pi_1} \cdot \gamma_1 h(\xi_1^*(\overline{c}_1)) + \frac{1-\pi_1}{\pi_1} \cdot \gamma_2 h(\xi_2^*(\underline{c}_2)) \right\}. \tag{B.26}$$

To be able to solve for $\gamma_2 h(\xi_2^*(\underline{c}_2))$, we assume for the moment that

$$1 - \alpha \frac{1-\pi_1}{\pi_1} > 0. \tag{B.27}$$

Then, rewriting (B.26), we obtain

$$\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \underbrace{\left\{ \frac{\alpha \cdot \frac{1}{\pi_1}}{1 - \alpha \cdot \frac{1-\pi_1}{\pi_1}} \right\}}_{\equiv \lambda} \cdot \gamma_1 h(\xi_1^*(\overline{c}_1)). \tag{B.28}$$

Thus, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \lambda \cdot \gamma_1 h(\xi_1^*(\overline{c}_1))$. We claim that inequality (B.27) holds. Indeed, starting with Assumption 1, we find that

$$\gamma < \frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1} \cdot \widehat{\sigma} \quad \Leftrightarrow \quad \frac{\gamma}{\widehat{\sigma}} + 1 < \frac{2\pi_2}{2-\pi_1} \tag{B.29}$$

$$\Leftrightarrow \quad \underbrace{\frac{(\gamma/\widehat{\sigma}) + 1}{\pi_2}}_{=\alpha+1} < \underbrace{\frac{2}{2-\pi_1}}_{=\frac{\pi_1}{2-\pi_1}+1} \tag{B.30}$$

$$\Leftrightarrow \quad \alpha < \frac{\pi_1}{2-\pi_1} \tag{B.31}$$

$$\Leftrightarrow \quad 1 - \frac{\alpha(1-\pi_1)}{\pi_1} > \frac{\alpha}{\pi_1}. \tag{B.32}$$

Clearly, this implies (B.27). Moreover, it can be readily verified that (B.32) implies $\lambda < 1$. Therefore, $\gamma_2 h(\xi_2^*(\underline{c}_2)) \leq \gamma_1 h(\xi_1^*(\overline{c}_1))$. Using the monotonicity of equilibrium bid schedules (Lemma B.2 above), this yields $\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1))$ for any $c_1 \in C_1$ and $c_2 \in C_2$. This proves property (ii) in Definition 1 for the case that all types of both players conceal their private information.

Case B. Supp$(\mu_i) \subsetneq C_i$ *for some player* $i \in \{1, 2\}$. The conclusion remains valid even if not all types conceal. To understand why, note that disclosure by some types means that, in the relevant information set at the contest stage, the sets C_1 and C_2 are replaced by nonempty subsets, respectively. Therefore, player 1's lowest relative resolve $\sigma = \underline{c}_2/\overline{c}_1$ rises weakly. Given that the curvature $\underline{\rho} \geq 1$ stays unchanged, this implies that $\widehat{\sigma}(\sigma, \underline{\rho})$ rises weakly as well. Further, player 1 and 2's predictabilities π_1 and π_2 fall weakly, while the net bias γ stays the same. Therefore, Assumption 1 continues to hold, and the argument detailed under case A goes through as before.

This concludes the proof of property (ii) of Definition 1. It remains to verify property (i) of the definition of uniform asymmetry, i.e., that all types of player 1 are active. Suppose not. Then, all types of player 2 are active. Denote by $\varnothing \neq C_1^* \subsetneq C_1$ the set of active types of player 1, and by $q_1^* = \sum_{c_1 \in C_1^*} q_1(c_1)$ the ex-ante probability that player 1 is active. Then, since any positive bid wins against an inactive type with probability one, the corresponding terms in player 2's first-order condition vanish, so that

$$\sum_{c_1 \in C_1^*} q_1(c_1) \frac{\gamma_2 h'(\xi_2^*(c_2))\gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = c_2 \qquad (c_2 \in C_2). \tag{B.33}$$

In the modified contest, player 1's type set C_1 is replaced by the subset C_1^*, the probability distribution $q_1(.)$ is replaced by $q_1^*(c_1) = q_1(c_1)/q_1^*$, and player 2's type set C_2 is replaced by $C_2/q_1^* = \{c_2/q_1^* | \, c_2 \in C_2\}$. Denote by $\xi_1^*|_{C_1^*}$ the restriction of the mapping $\xi_1^* : C_1 \to \mathbb{R}_+$ to C_1^*, and by $\xi_2^*|_{q_1^*} : \frac{C_2}{q_1^*} \to \mathbb{R}_+$ the bid schedule for player 2 in the modified contest that satisfies $\xi_2^*|_{q_1^*}(\frac{c_2}{q_1^*}) = \xi_2^*(c_2)$ for any $c_2 \in C_2$. We claim that $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is a Bayesian equilibrium in the modified contest. Indeed, the first-order condition of any active type of player 1 holds in the modified contest. Moreover, dividing (B.33) by $q_1^* > 0$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(\xi_2^*(c_2))\gamma_1 h(\xi_1^*(c_1))}{(\gamma_1 h(\xi_1^*(c_1)) + \gamma_2 h(\xi_2^*(c_2)))^2} = \frac{c_2}{q_1^*} \qquad (c_2 \in C_2), \tag{B.34}$$

i.e., also the first-order condition of any type of player 2 holds in the modified contest. Since all types of both players are active in $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ and since, in addition, the expected payoff against a player that is always active is strictly concave in the own bid, this proves the claim, i.e., $(\xi_1^*|_{C_1^*}, \xi_2^*|_{q_1^*})$ is indeed a Bayesian equilibrium in the modified contest. Next, one notes that, since Assumption 1 holds for the original contest, Assumption 1 holds also for the modified contest (because π_1 and σ rise weakly, while γ, $\underline{\rho}$, and π_2 stay the same). From the first part of the proof, applied to the modified contest, it therefore follows that

$$\gamma_2 h(\xi_2^*(c_2)) \leq \gamma_1 h(\xi_1^*(c_1)) \qquad (c_1 \in C_1^*, c_2 \in C_2). \tag{B.35}$$

Now, by assumption, some types of player 1 remain inactive in the original contest. Since, by Lemma B.2, ξ_1^* is monotone declining, this clearly implies $\xi_1^*(\overline{c}_1) = 0$. Consequently, the marginal productivity at the zero bid level $h'(0) = \lim_{\varepsilon \searrow 0} \frac{h(\varepsilon)}{\varepsilon}$ is finite. Moreover, type $\overline{c}_1$'s marginal payoff at the zero bid level is weakly negative, i.e.,

$$E_{c_2}\left[\frac{\gamma_1 h'(0)}{\gamma_2 h(\xi_2^*(c_2))}\right] \leq \overline{c}_1. \tag{B.36}$$

Plugging (B.35) into (B.36), we see that

$$\frac{h'(0)}{h(\xi_1^*(c_1))} \leq \overline{c}_1 \qquad (c_1 \in C_1^*). \tag{B.37}$$

Moreover, Assumption 1 implies

$$\frac{\gamma_2}{\gamma_1} = \gamma < \underbrace{\frac{\pi_1 + 2\pi_2 - 2}{2 - \pi_1}}_{\leq 1} \cdot \underbrace{\widehat{\sigma}(\sigma, \underline{\rho})}_{\leq \sigma} \leq \sigma = \frac{\underline{c}_2}{\overline{c}_1}. \tag{B.38}$$

Multiplying inequality (B.37) by $(\gamma/q_1^*) > 0$, exploiting (B.38), and taking expectations over all $c_1 \in C_1^*$, we get

$$\sum_{c_1 \in C_1^*} \frac{q_1(c_1)}{q_1^*} \frac{\gamma_2 h'(0)}{\gamma_1 h(\xi_1^*(c_1))} < \frac{\underline{c}_2}{q_1^*}. \tag{B.39}$$

Thus, in the modified contest, the marginal expected payoff of type $(\underline{c}_2/q_1^*)$ at the zero bid level is strictly negative. But this is impossible given that she is active and her expected payoff against $\xi_1^*|_{C_1^*}$ is strictly concave. The contradiction shows that, indeed, all types of player 1 are active in the original contest. □

C. Material omitted from Section 4

This section contains two auxiliary results, proofs of Propositions 1&2 and Theorem 1, as well as some discussion.

C.1 Best-response monotonicity

We will say that *player 1's domain condition* holds at $(\xi_2; c_1) \in X_2^* \times C_1$ if (i) $\widetilde{\beta}_1(\xi_2; c_1) > 0$, and (ii) $p_1(\widetilde{\beta}_1(\xi_2; c_1), \xi_2(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$. Thus, player 1's domain condition at $(\xi_2; c_1)$ requires that type c_1's best-response bid against ξ_2 is interior, and wins with a probability strictly exceeding one half against any of player 2's types. Similarly, we will say that *player 2's domain condition* holds at $(\widehat{\xi}_1; c_2) \in X_1^* \times C_2$ if (i) $\widetilde{\beta}_2(\widehat{\xi}_1; c_2) > 0$, and (ii) $p_2(\widetilde{\beta}_2(\widehat{\xi}_1; c_2), \widehat{\xi}_1(c_1)) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, player 2's domain condition at $(\widehat{\xi}_1; c_1)$ requires that type c_2's best-response bid against $\widehat{\xi}_1$ is interior, and wins with a probability strictly below one half against any of player 1's types.

Lemma C.1 (Best-response monotonicity)

(i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and let $c_1 \in C_1$. If player 1's domain condition holds at $(\xi_2; c_1)$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$. In particular, if player 1's domain condition holds at $(\xi_2; c_1)$ for every $c_1 \in C_1$, then $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$.

(ii) Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and let $c_2 \in C_2$. If player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$, then $\widetilde{\beta}_2(\xi_1; c_2) < \widetilde{\beta}_2(\widehat{\xi}_1; c_2)$. In particular, if player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$ for every $c_2 \in C_2$, then $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$.

Proof. (i) Let $\xi_2, \widehat{\xi}_2 \in X_2^*$ with $\xi_2 \succ \widehat{\xi}_2$, and $c_1 \in C_1$. By assumption, player 1's domain condition holds at $(\xi_2; c_1)$. We wish to show that $x_1 \equiv \widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1) \equiv \widehat{x}_1$. To provoke a contradiction, suppose that $\widehat{x}_1 \geq x_1$. From the domain condition, we have $x_1 > 0$. Therefore, both x_1 and $\widehat{x}_1$ are positive, so that the corresponding first-order conditions imply

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] = E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right] = c_1. \tag{C.1}$$

Fix some $c_2 \in C_2$ for the moment. Letting $x = \gamma_1 h(\widetilde{\beta}_1(\xi_2; c_1))$ and $y = \gamma_2 h(\xi_2(c_2))$, the domain condition implies $x > y$. Clearly, the mapping $y \mapsto y/(x+y)^2$ is strictly increasing over the interval $[0, x]$. Therefore, noting that $\xi_2 \succ \widehat{\xi}_2$ implies $y \geq \widehat{y} \equiv \gamma_2 h(\widehat{\xi}_2(c_2))$, we see that

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2} \geq \frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2), \tag{C.2}$$

with strict inequality for at least one $c_2 \in C_2$. Moreover, from $\widehat{x}_1 \geq x_1$,

$$\frac{\gamma_1 h'(x_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \geq \frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2} \qquad (c_2 \in C_2). \tag{C.3}$$

Combining (C.2) and (C.3), and subsequently taking expectations, we arrive at

$$E_{c_2}\left[\frac{\gamma_1 h'(x_1)\gamma_2 h(\xi_2(c_2))}{(\gamma_1 h(x_1) + \gamma_2 h(\xi_2(c_2)))^2}\right] > E_{c_2}\left[\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{\xi}_2(c_2))}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{\xi}_2(c_2)))^2}\right], \tag{C.4}$$

in conflict with (C.1). The contradiction shows that $x_1 > \widehat{x}_1$, as claimed. Moreover, if player 1's domain condition holds for any $c_1 \in C_1$, then $\widetilde{\beta}_1(\xi_2; c_1) > \widetilde{\beta}_1(\widehat{\xi}_2; c_1)$ for any $c_1 \in C_1$, which indeed implies $\beta_1(\xi_2) \succ \beta_1(\widehat{\xi}_2)$. (ii) The proof is similar. Let $\xi_1, \widehat{\xi}_1 \in X_1^*$ with $\xi_1 \succ \widehat{\xi}_1$, and $c_2 \in C_2$. By assumption, player 2's domain condition holds at $(\widehat{\xi}_1; c_2)$. Suppose that $x_2 \equiv \widetilde{\beta}_2(\xi_1; c_2) \geq \widetilde{\beta}_2(\widehat{\xi}_1; c_2) \equiv \widehat{x}_2$. Then, from the domain condition, $\widehat{x}_2 > 0$. Hence,

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] = E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right] = c_2. \tag{C.5}$$

Fix some $c_1 \in C_1$, and let $\widehat{x} = \gamma_2 h(\widetilde{\beta}_2(\widehat{\xi}_1; c_2))$ and $\widehat{y} = \gamma_1 h(\widehat{\xi}_1(c_1))$. By the domain condition, $\widehat{x} < \widehat{y}$. Moreover, the mapping $\widehat{y} \mapsto \widehat{y}/(\widehat{x}+\widehat{y})^2$ is strictly declining for $\widehat{y} \geq \widehat{x}$. Hence, given that $\widehat{\xi}_1 \prec \xi_1$ implies $\widehat{y} \leq y \equiv \gamma_1 h(\xi_1(c_1))$, we see that

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2} \geq \frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(\widehat{x}_2))^2} \qquad (c_1 \in C_1), \tag{C.6}$$

with strict inequality for some $c_1 \in C_1$. Moreover, from $\widehat{x}_2 \leq x_2$,

$$\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_2)) + \gamma_2 h(\widehat{x}_2))^2} \geq \frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_2 h(\xi_1(c_1)) + \gamma_2 h(x_1))^2} \qquad (c_1 \in C_1). \tag{C.7}$$

Combining (C.6) and (C.7), and taking expectations, we arrive at

$$E_{c_1}\left[\frac{\gamma_2 h'(\widehat{x}_2)\gamma_1 h(\widehat{\xi}_1(c_1))}{\left(\gamma_1 h(\widehat{\xi}_1(c_1)) + \gamma_2 h(\widehat{x}_2)\right)^2}\right] > E_{c_1}\left[\frac{\gamma_2 h'(x_2)\gamma_1 h(\xi_1(c_1))}{(\gamma_1 h(\xi_1(c_1)) + \gamma_2 h(x_2))^2}\right], \tag{C.8}$$

in contradiction to (C.5). It follows that, indeed, $\widehat{x}_2 > x_2$. In particular, provided that player 2's domain condition holds for any $c_2 \in C_2$, it follows that $\beta_2(\xi_1) \prec \beta_2(\widehat{\xi}_1)$. This concludes the proof. □

Lemma C.1 shows that the domain conditions are sufficient to ensure that a type's best-response bid and a player's best-response bid schedule, respectively, move in a strictly monotone way to changes in the opponent's bid schedule. For example, in the case of player 1, the best-response bid of type c_1 will strictly rise in response to an increase of player 2's bid schedule. If player 1's domain condition holds at all of her types, then we get a strict order relation even between the best-response bid schedules. Similar comparative statics properties hold for player 2, whose best-response mapping is, however, strictly declining under the assumptions of Lemma C.1. In sum, the contest with two-sided incomplete information exhibits, subject to domain conditions, comparative statics properties analogous to those of the complete-information contest.

C.2 Stackelberg monotonicity

The next auxiliary result establishes monotonicity properties of the complete-information contest.

Lemma C.2 (Stackelberg monotonicity) *Let $x_2 > \widehat{x}_2 \geq 0$ and $c_1 \in C_1$ such that $x_1 = \widetilde{\beta}_1(\psi_2(x_2); c_1)$ and $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1)$. If $\widehat{x}_1 > 0$, then (i) $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$ and (ii) $\Pi_1(x_1, x_2; c_1) < \Pi_1(\widehat{x}_1, \widehat{x}_2; c_1)$.*

Proof. (i) By assumption, $\widehat{x}_1 = \widetilde{\beta}_1(\psi_2(\widehat{x}_2); c_1) > 0$. Therefore, $x_2 > \widehat{x}_2$ implies $p_2(\widehat{x}_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$. Assume first that $x_1 \leq \widehat{x}_1$. Then, clearly, $p_2(x_1, x_2) \geq p_2(\widehat{x}_1, x_2)$ and, hence, $p_2(x_1, x_2) > p_2(\widehat{x}_1, \widehat{x}_2)$, as claimed. Assume next that $x_1 > \widehat{x}_1$. Then, the necessary first-order conditions associated with the respective optimality of $\widehat{x}_1$ and x_1 hold true. As for $\widehat{x}_1$, we find that

$$\frac{\gamma_1 h'(\widehat{x}_1)\gamma_2 h(\widehat{x}_2)}{(\gamma_1 h(\widehat{x}_1) + \gamma_2 h(\widehat{x}_2))^2} = c_1. \tag{C.9}$$

Multiplying by $\gamma h(\widehat{x}_2)/h'(\widehat{x}_1)$, with $\gamma = \gamma_2/\gamma_1$ as before, yields $(p_2(\widehat{x}_1, \widehat{x}_2))^2 = c_1\gamma h(\widehat{x}_2)/h'(\widehat{x}_1)$. Similarly, one shows that the optimality of x_1 implies $(p_2(x_1,x_2))^2 = c_1\gamma h(x_2)/h'(x_1)$. Recalling that h is strictly increasing and that h' is weakly declining, we see that $(p_2(x_1,x_2))^2 > (p_2(\widehat{x}_1,\widehat{x}_2))^2$. The claim follows. (ii) As a consequence of the envelope theorem,

$$\frac{d\Pi_1(\widetilde{\beta}_1(\psi_2(x_2);c_1),x_2;c_1)}{dx_2} = \frac{\partial\Pi_1(x_1,x_2;c_1)}{\partial x_2}\bigg|_{x_1=\widetilde{\beta}_1(\psi_2(x_2);c_1)} = -\frac{\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2);c_1))\gamma_2 h'(x_2)}{(\gamma_1 h(\widetilde{\beta}_1(\psi_2(x_2);c_1))+\gamma_2 h(x_2))^2} < 0. \quad \text{(C.10)}$$

Thus, player 1 indeed strictly benefits from the lowered effort of player 2. This proves the second claim and, hence, the lemma. □

C.3 Proof of Proposition 1

The following proof establishes the strict incentive of the weakest type of the underdog to self-disclose.

Proof of Proposition 1. The conclusions of Proposition 1 are immediate if $\xi_2^*(\overline{c}_2) = 0$. Suppose that $\xi_2^*(\overline{c}_2) > 0$. Since, by Lemma B.2, the equilibrium bid schedule ξ_2^* is weakly declining, actually all types of player 2 are active in ξ_2^*. Using Lemma B.2 another time, one sees that ξ_2^* is even strictly declining. These observations will be tacitly used below. We now prove the three assertions made in the statement of the proposition. (i) First, it is shown that self-disclosure induces the weakest type of the underdog to strictly raise her bid, i.e., $\xi_2^*(\overline{c}_2) < x_2^{\#}$. To provoke a contradiction, suppose that $\xi_2^*(\overline{c}_2) \geq x_2^{\#}$. Then, because ξ_2^* is strictly declining and there are at least two possible type realizations for player 2, we get $\xi_2^* \succ \psi_2(x_2^{\#})$. We claim that player 1's domain condition holds at $(\xi_2^*;c_1)$, for any $c_1 \in C_1$. To see this, take some $c_1 \in C_1$. Then, from property (i) of uniform asymmetry, $\widetilde{\beta}_1(\xi_2^*;c_1) = \xi_1^*(c_1) > 0$. Further, since all types of player 2 are active in ξ_2^*, property (ii) of uniform asymmetry implies that $p_1(\xi_1^*(c_1),\xi_2^*(c_2)) > \frac{1}{2}$ for any $c_2 \in C_2$, which proves the claim. We may, therefore, apply Lemma C.1(i) so as to obtain

$$\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(x_2^{\#})) = \xi_1^{\#}. \quad \text{(C.11)}$$

Next, it is claimed that player 2's domain condition holds at $(\xi_1^{\#};\overline{c}_2)$. Since $(\xi_1^{\#}(.),x_2^{\#})$ is an equilibrium in the contest with one-sided incomplete information, we have $x_2^{\#} > 0$, i.e., player 2 is active with probability one. Invoking property (ii) of uniform asymmetry shows, therefore, that $p_2(\xi_1^{\#}(c_1),x_2^{\#}) < \frac{1}{2}$ holds true for any $c_1 \in C_1$. Since $\widetilde{\beta}_2(\xi_1^{\#};\overline{c}_2) = x_2^{\#}$, this means that $p_2(\xi_1^{\#}(c_1),\widetilde{\beta}_2(\xi_1^{\#};\overline{c}_2)) < \frac{1}{2}$, for any $c_1 \in C_1$. I.e., player 2's domain condition at $(\xi_1^{\#};\overline{c}_2)$ is indeed satisfied. Therefore, using relationship (C.11) and Lemma C.1(ii), we see that $\xi_2^*(\overline{c}_2) = \widetilde{\beta}_2(\xi_1^*;\overline{c}_2) < \widetilde{\beta}_2(\xi_1^{\#};\overline{c}_2) = x_2^{\#}$, in contradiction to $\xi_2^*(\overline{c}_2) \geq x_2^{\#}$. Thus, $\xi_2^*(\overline{c}_2) < x_2^{\#}$, as claimed. (ii) Next, it is shown that, after disclosure, the probability of winning for the weakest type of the underdog rises strictly, i.e., $p_2^{\#} = E_{c_1}[p_2(x_2^{\#},\xi_1^{\#}(c_1))] > E_{c_1}[p_2(\xi_2^*(\overline{c}_2),\xi_1^*(c_1))] = p_2^*$. In fact, we will prove the somewhat stronger statement

$$p_2(x_2^{\#},\xi_1^{\#}(c_1)) > p_2(\xi_2^*(\overline{c}_2),\xi_1^*(c_1)) \qquad (c_1 \in C_1). \quad \text{(C.12)}$$

Take some type $c_1 \in C_1$. It is claimed first that $\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2));c_1) > 0$, as shown in the left diagram of Figure C.1. Indeed, because player 2 is always active in ξ_2^*, the mapping $x_1 \mapsto E_{c_2}[\Pi_1(x_1,\xi_2^*(c_2);c_1)]$ is strictly concave on $\mathbb{R}_+$, and vanishes at $x_1 = 0$. Therefore, the optimality of $\xi_1^*(c_1) > 0$ implies $E_{c_2}[\Pi_1(\xi_1^*(c_1),\xi_2^*(c_2);c_1)] > 0$. But the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$ is everywhere weakly lower than ξ_2^*. Therefore, $E_{c_2}[\Pi_1(\xi_1^*(c_1),\psi_2(\xi_2^*(\overline{c}_2));c_1)] > 0$, i.e., type c_1 is able to realize a positive payoff against the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$. Since $\xi_2^*(\overline{c}_2) > 0$, it follows that type c_1's best-response bid against $\psi_2(\xi_2^*(\overline{c}_2))$ is positive, as claimed. Next, from the previous step, we know that $x_2^{\#} > \xi_2^*(\overline{c}_2)$. Invoking Lemma C.2(i), and noting that $\xi_1^{\#} = \beta_1(\psi_2(x_2^{\#}))$, it follows that

$$p_2(x_2^{\#},\xi_1^{\#}(c_1)) > p_2(\xi_2^*(\overline{c}_2),\widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2));c_1)) \qquad (c_1 \in C_1). \quad \text{(C.13)}$$

Next, comparing the strictly declining equilibrium bid schedule $\xi_2^* = \beta_2(\xi_1^*)$ with the flat bid schedule $\psi_2(\xi_2^*(\overline{c}_2))$, and recalling that there are at least two types, we obtain $\xi_2^* \succ \psi_2(\xi_2^*(\overline{c}_2))$. Moreover, as seen above, all types of player 2 are active. Hence, by property (ii) of uniform asymmetry, $p_1(\xi_1^*(c_1),\xi_2^*(c_2)) > \frac{1}{2}$ for any $c_1 \in C_1$ and any $c_2 \in C_2$, so that via $\widetilde{\beta}_1(\xi_2^*;c_1) = \xi_1^*(c_1)$, player 1's domain condition is seen to hold at $(\xi_2^*;c_1)$, for any

$c_1 \in C_1$. Therefore, by Lemma C.1(i), $\xi_1^* = \beta_1(\xi_2^*) \succ \beta_1(\psi_2(\xi_2^*(\overline{c}_2)))$, as illustrated in Figure C.1.[2] In particular, $\xi_1^*(c_1) \geq \widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1)$, for any $c_1 \in C_1$. Therefore,

$$p_2(\xi_2^*(\overline{c}_2), \widetilde{\beta}_1(\psi_2(\xi_2^*(\overline{c}_2)); c_1)) \geq p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1)) \qquad (c_1 \in C_1). \tag{C.14}$$

Combining (C.13) and (C.14) yields (C.12). In particular, this proves $p_2^\# > p_2^*$, as claimed.

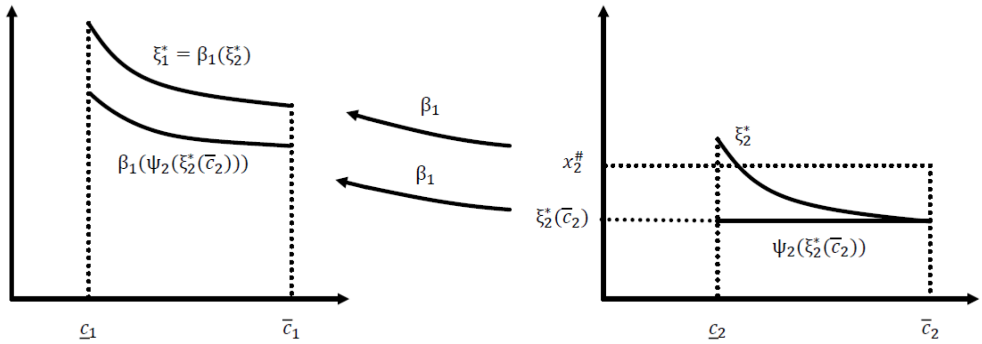


Figure C.1 Proof of Proposition 1(ii).

(iii) Finally, we show that the weakest type of the underdog has a strict incentive to disclose her type. Clearly, the equilibrium effort $x_2^\#$ is positive. One can check that type $\overline{c}_2$'s first-order condition is equivalent to

$$E_{c_1}\left[p_2(x_2^\#, \xi_1^\#(c_1)) - \left(p_2(x_2^\#, \xi_1^\#(c_1))\right)^2\right] = \overline{c}_2 \Phi(x_2^\#). \tag{C.15}$$

Exploiting (C.15), we obtain for type $\overline{c}_2$'s expected payoff from self-disclosure,

$$\Pi_2^\# = E_{c_1}\left[\left(p_2(x_2^\#, \xi_1^\#(c_1))\right)^2\right] + \overline{c}_2\left(\Phi(x_2^\#) - x_2^\#\right). \tag{C.16}$$

In a completely analogous fashion, we can convince ourselves that concealment grants type $\overline{c}_2$ a payoff of

$$\Pi_2^*(\overline{c}_2) = E_{c_1}\left[(p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1)))^2\right] + \overline{c}_2\left(\Phi(\xi_2^*(\overline{c}_2)) - \xi_2^*(\overline{c}_2)\right). \tag{C.17}$$

Now, from (C.12), we see that $E_{c_1}[(p_2(x_2^\#, \xi_1^\#(c_1)))^2] > E_{c_1}[(p_2(\xi_2^*(\overline{c}_2), \xi_1^*(c_1)))^2]$. Moreover, from Lemma B.1, $\Phi' \geq 1$, so that the mapping $x_2 \mapsto \Phi(x_2) - x_2$ is monotone increasing in x_2. But, as shown above, $\xi_2^*(\overline{c}_2) < x_2^\#$. It follows that the weakest type of the underdog has indeed a strict incentive to reveal her type. This proves the final claim and concludes the proof of the proposition. □

C.4 Proof of Proposition 2

Next, we present the proof of Proposition 2, regarding the strict incentive of the strongest type of the favorite to reveal her type, provided that the underdog's type is public information.

Proof of Proposition 2. Since x_1° and x_2° are equilibrium efforts under complete information, we have $x_1^\circ > 0$ and $x_2^\circ > 0$. Similarly, one notes that $x_2^\# > 0$. Moreover, by property (i) of uniform asymmetry, all types of player 1 are active in $\xi_1^\#$, so that by Lemma B.2, the bid schedule $\xi_1^\#$ is strictly declining. We now prove the four assertions made in the statement of Proposition 2. (i) It is claimed that $x_2^\circ < x_2^\#$. To provoke a contradiction, suppose that $x_2^\circ \geq x_2^\#$. Part (ii) of uniform asymmetry implies $p_1(x_1^\circ, x_2^\circ) > \frac{1}{2}$, so that in view of $x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1)$,

[2] The figure shows an example where $x_2^\# < \xi_2^*(\underline{c}_2)$. In general, we may also have that $x_2^\# \geq \xi_2^*(\underline{c}_2)$.

player 1's domain condition holds at $(x_2^\circ; \underline{c}_1)$. Hence, by Lemma C.1(i), if even $x_2^\circ > x_2^\#$, then $x_1^\circ = \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) > \widetilde{\beta}_1(x_2^\#; \underline{c}_1) = \xi_1^\#(\underline{c}_1)$. If, however, $x_2^\circ = x_2^\#$, then it is immediate that $x_1^\circ = \xi_1^\#(\underline{c}_1)$. Thus, either way, we arrive at $x_1^\circ \geq \xi_1^\#(\underline{c}_1)$, so that $\psi_1(x_1^\circ) \succeq \psi_1(\xi_1^\#(\underline{c}_1))$. Moreover, given that player 1 has at least two types, and that $\xi_1^\#$ is strictly declining, $\psi_1(\xi_1^\#(\underline{c}_1)) \succ \xi_1^\#$. Hence, $\psi_1(x_1^\circ) \succ \xi_1^\#$. Part (ii) of uniform asymmetry implies that $p_2(\xi_1^\#(c_1), x_2^\#) < \frac{1}{2}$ for any $c_1 \in C_1$. Thus, recalling that $x_2^\# = \widetilde{\beta}_2(\xi_1^\#; c_2^\#)$, player 2's domain condition holds at $(\xi_1^\#; c_2^\#)$. Therefore, using Lemma C.1(ii), we arrive at $x_2^\# = \widetilde{\beta}_2(\xi_1^\#; c_2^\#) > \widetilde{\beta}_2(\psi_1(x_1^\circ); c_2^\#) = x_2^\circ$, a contradiction. It follows that $x_2^\circ < x_2^\#$, as claimed. (ii) Next, it is shown that $x_1^\circ < \xi_1^\#(\underline{c}_1)$. From the previous step, we know that $x_2^\# > x_2^\circ$. Via property (ii) of uniform asymmetry, we see that $p_1(\xi_1^\#(\underline{c}_1), x_2^\#) > \frac{1}{2}$. Thus, the domain condition for player 1 holds at $(x_2^\#; \underline{c}_1)$. Lemma C.1(i) implies, therefore, that $\xi_1^\#(\underline{c}_1) = \widetilde{\beta}_1(x_2^\#; \underline{c}_1) > \widetilde{\beta}_1(x_2^\circ; \underline{c}_1) = x_1^\circ$. Thus, the effort of the strongest type of the favorite will indeed be strictly lower after self-disclosure. (iii) Given part (i) above, we have $x_2^\circ < x_2^\#$. Recalling that $x_1^\circ > 0$, Lemma C.2(i) implies $p_2(x_1^\circ, x_2^\circ) < p_2(\xi_1^\#(\underline{c}_1), x_2^\#)$, so that $p_1(x_1^\circ, x_2^\circ) > p_1(\xi_1^\#(\underline{c}_1), x_2^\#)$. Thus, type $\underline{c}_1$ indeed wins with a strictly higher probability after self-disclosure. (iv) The claim that $\Pi_1^\circ > \Pi_1^\#$ follows now directly from Lemma C.2(ii). This completes the proof. □

C.5 Proof of Theorem 1

This subsection combines the auxiliary results to prove our main result.

Proof of Theorem 1. We start by showing that self-disclosure by all types of both players constitutes a perfect Bayesian equilibrium. To this end, we specify off-equilibrium beliefs $\mu_1^0 \in \Delta(C_1)$ and $\mu_2^0 \in \Delta(C_2)$ as follows. The underdog expects a favorite that does not disclose her private information to be of type $c_1 = \overline{c}_1$ with probability one. Thus, $\mu_1^0(c_1) = 1$ if $c_1 = \overline{c}_1$, and $\mu_1^0(c_1) = 0$ otherwise. Similarly, the favorite expects an underdog that does not disclose her private information to be of type $c_2 = \underline{c}_2$ with probability one. Thus, $\mu_2^0(c_2) = 1$ if $c_2 = \underline{c}_2$, and $\mu_2^0(c_2) = 0$ otherwise. To check the equilibrium property, consider first an arbitrary type $c_1 \in C_1$ of the favorite. If c_1 complies with equilibrium self-disclosure and is matched with some type $c_2 \in C_2$ of the underdog, then c_1 receives a complete-information equilibrium payoff of $\Pi_1^\circ(c_1, c_2) = \Pi_1(\widetilde{\beta}_1(x_2^\circ(c_1, c_2); c_1), x_2^\circ(c_1, c_2); c_1)$. If, however, c_1 chooses to not disclose then, given the off-equilibrium beliefs specified above, an underdog of type c_2 expects the favorite to be of the worst-case type $\overline{c}_1$ and, having revealed her own type c_2, chooses an effort of $x_2^\circ(\overline{c}_1, c_2)$. Responding optimally to type c_2's bid, the deviating favorite of type c_1 chooses an effort of $\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1)$ at the contest stage, and consequently receives a payoff of $\Pi_1^{\text{dev}}(c_1, c_2) = \Pi_1(\widetilde{\beta}_1(x_2^\circ(\overline{c}_1, c_2); c_1), x_2^\circ(\overline{c}_1, c_2); c_1)$. A straightforward application of Monaco and Sabarwal (2016, Thm. 3) shows that, given Assumption 1, $x_2^\circ(c_1, c_2) \leq x_2^\circ(\overline{c}_1, c_2)$.[3] We claim that $\Pi_1^\circ(c_1, c_2) \geq \Pi_1^{\text{dev}}(c_1, c_2)$. Indeed, if $x_2^\circ(c_1, c_2) < x_2^\circ(\overline{c}_1, c_2)$ then, by Lemma C.2(ii), $\Pi_1^\circ(c_1, c_2) > \Pi_1^{\text{dev}}(c_1, c_2)$. Moreover, if $x_2^\circ(c_1, c_2) = x_2^\circ(\overline{c}_1, c_2)$ then $\Pi_1^\circ(c_1, c_2) = \Pi_1^{\text{dev}}(c_1, c_2)$, which proves the claim. Taking expectations over all $c_2 \in C_2$ yields $E_{c_2}[\Pi_1^\circ(c_1, c_2)] \geq E_{c_2}[\Pi_1^{\text{dev}}(c_1, c_2)]$, for any $c_1 \in C_1$. Hence, a deviation is not profitable for any type $c_1 \in C_1$. On the other hand, if any type of the underdog deviates, and the favorite interprets this as a tactic of the strongest type of the underdog, then one shows in complete analogy that the equilibrium condition holds. It follows that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. Next, suppose there is a perfect Bayesian equilibrium in which not all private information is revealed. Then, for at least one player $i \in \{1, 2\}$, the set of types concealing their signal, $C_i \backslash S_i$, has at least two elements. By suitably redefining C_1 and C_2, we may assume without loss of generality that all types conceal their types. Suppose first that $K_2 \geq 2$. Then, Proposition 1 implies that the weakest type of the underdog has a strict incentive to unilaterally deviate at the revelation stage, in conflict with the equilibrium assumption. Suppose next that $K_2 = 1$. Then, since there is incomplete information, $K_1 \geq 2$. But, again, this cannot be part of a perfect Bayesian equilibrium by Proposition 2. Thus, either way, we obtain a contradiction, and the claim follows. This proves the theorem. □

C.6 Discussion: Dominance and defiance

To make transparent why analyzing disclosure in probabilistic contests requires new methods, we discuss the comparative statics of the Bayesian equilibrium at the contest stage with respect to changes in the information structure.

First, while self-disclosure by the weakest type of the underdog tends to have an overall moderating effect on the favorite, some types of the favorite may respond by bidding higher.

[3] For a self-contained argument, it suffices to replicate earlier arguments. Indeed, suppose that $x_2^\circ(c_1, c_2) > x_2^\circ(\overline{c}_1, c_2)$. Clearly, all equilibrium efforts are positive under complete information. Therefore, using property (ii) of uniform asymmetry, player 1's domain condition holds at $(x_2^\circ(c_1, c_2); c_1)$, so that, by Lemma C.1(i), $x_1^\circ(c_1, c_2) > x_1^\circ(\overline{c}_1, c_2)$. Moreover, using property (ii) of uniform asymmetry another time, player 2's domain condition is seen to hold at $(x_1^\circ(\overline{c}_1, c_2); c_2)$, so that by Lemma C.1(ii), $x_2^\circ(c_1, c_2) < x_2^\circ(\overline{c}_1, c_2)$, which yields the desired contradiction.

Example C.1 ("Dominant reaction")[4] Table C.1 exhibits data for a uniformly asymmetric contest. As can be seen, after the self-disclosure by $\overline{c}_2 = c_2^2$, the weak type of the favorite, $\overline{c}_1 = c_1^2$, raises her effort.

Player 1		Player 2	
$c_1^1 = 0.1$	$c_1^2 = 0.2$	$c_2^1 = 3.7$	$c_2^2 = 7.8$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.05$	$q_2(c_2^2) = 0.95$
$\xi_1^*(c_1^1) = 0.1592$	$\xi_1^*(c_1^2) = 0.1042$	$\xi_2^*(c_2^1) = 0.0572$	$\xi_2^*(c_2^2) = 0.0011$
$\vee$	$\wedge$		$\wedge$
$\xi_1^{\#}(c_1^1) = 0.1495$	$\xi_1^{\#}(c_1^2) = 0.1051$	–	$x_2^{\#} = 0.0023$

Table C.1 Equilibrium bids before and after the underdog's self-disclosure.

Despite the non-monotonicity illustrated by the example, the model does impose some structure on the favorite's reaction. First, not all types of the favorite may simultaneously raise their bids in response to the self-disclosure by the weakest type of the underdog. Indeed, this would be incompatible with our earlier conclusion that the weakest type of the underdog necessarily raises her bid. Second, even a dominant reaction of the favorite will never be strong enough to push the probability of winning for the weakest type of the underdog weakly below her probability of winning under concealment.

In analogy to the case just considered, a relatively strong type of the underdog may raise her effort in response to the favorite's attempt to discourage her.

Example C.2 ("Defiant reaction") Consider the uniformly asymmetric contest characterized in Table C.2. In response to the attempt by the strongest type of the favorite to discourage the underdog, only the two weaker types of the underdog lower their respective efforts, whereas the strongest type of the underdog raises her effort. In fact, the example illustrates another possibility mentioned in the body of the paper, viz. that a type of the underdog may decide to exert zero effort.

Player 1		Player 2		
$c_1^1 = 0.2$	$c_1^2 = 0.6$	$c_2^1 = 7.5$	$c_2^2 = 11$	$c_2^3 = 11.5$
$q_1(c_1^1) = 0.5$	$q_1(c_1^2) = 0.5$	$q_2(c_2^1) = 0.1$	$q_2(c_2^2) = 0.1$	$q_2(c_2^3) = 0.8$
$\xi_1^*(c_1^1) = 0.1195$	$\xi_1^*(c_1^2) = 0.0647$	$\xi_2^*(c_2^1) = 0.0205$	$\xi_2^*(c_2^2) = 0.0031$	$\xi_2^*(c_2^3) = 0.0014$
$\vee$		$\wedge$	$\vee$	$\vee$
$x_1^{\#} = 0.0872$	–	$\xi_2^{\#}(c_2^1) = 0.0206$	$\xi_2^{\#}(c_2^2) = 0.0018$	$\xi_2^{\#}(c_2^3) = 0.0000$

Table C.2 Equilibrium bids before and after the favorite's self-disclosure.

C.7 Games of strategic heterogeneity

In parameterized games of strategic heterogeneity (Monaco and Sabarwal, 2016; Barthel and Hoffmann, 2019), strategy spaces are multi-dimensional, and payoff functions allow for strategic complements and substitutes at the same time. Under suitable constraints on bids, the incomplete-information contests considered in the present paper would indeed satisfy the definition. Moreover, the monotone comparative statics of the contest stage with respect to changes in the information structure conducted above clearly draws on intuitions suggested by that literature. Quite notably, however, existing conditions do not apply to our model. As Examples C.1 and C.2 have shown, the relevant comparative statics of the Bayesian equilibrium is, in general, monotone for one player only. In contrast, Monaco and Sabarwal's (2016) conditions, like any of the conditions in the literature that we are aware of, imply the monotone comparative statics of the entire equilibrium profile. In fact, the contraction-mapping approach underlying Monaco and Sabarwal's (2016, Thm. 5) result need not go through when the contest is too asymmetric. The problem is that, as noted by Wärneryd (2018) in a different context, the iteration of the best response in an asymmetric contest with complete information need not be a contraction. Indeed, for low bids of the opponent, the best-response function in a probabilistic contest is very steep. The situation is similar under incomplete information. The following numerical example shows that monotone comparative statics results available for games of strategic heterogeneity do not apply to Example C.1.

[4] Unless stated otherwise, all numerical examples are based on the unbiased lottery contest.

Example C.1 (continued) Let $\overline{\beta}_1(\xi_2) = \beta_1(\psi_2(\xi_2(\overline{c}_2)))$ denote player 1's best-response bid schedule against $\psi_2(\xi_2(\overline{c}_2))$, where $\xi_2 \in X_2^*$. Monaco and Sabarwal (2016, Thm. 5) required that $\overline{\beta}_1(\widehat{\xi}_2) \preceq \xi_1^*$, where $\widehat{\xi}_2 = \beta_2(\widehat{\xi}_1)$ and $\widehat{\xi}_1 = \overline{\beta}_1(\xi_2^*)$. A computation shows that $\widehat{\xi}_1(\underline{c}_1) = 0.1016$, $\widehat{\xi}_1(\overline{c}_1) = 0.0715$, and $\widehat{\xi}_2(\overline{c}_2) = 0.0194$. As a result, $\overline{\beta}_1(\widehat{\xi}_2)(\underline{c}_1) = 0.4208 > 0.1592 = \xi_1^*(\underline{c}_1)$ and $\overline{\beta}_1(\widehat{\xi}_2)(\overline{c}_1) = 0.2919 > 0.1042 = \xi_1^*(\overline{c}_1)$. It follows that $\overline{\beta}_1(\widehat{\xi}_2) \succ \xi_1^*$, in conflict with the required condition.

D. Material omitted from Section 5

We go over the extensions discussed in the body of the paper.

D.1 Correlated types

If types are correlated, then each type $c_j \in C_j$ expects to face type c_i^k with a conditional probability $q_i(c_i^k\,|c_j\,) > 0$ for $k \in \{1, \ldots, K_i\}$. Expected payoffs are conditional expectations, and the definition of Bayesian equilibrium needs to be adapted correspondingly. We start with a particularly clean case in which types are negatively correlated.

Proposition D.1 (Negative correlation) *Suppose that Assumption 1 holds and that the conditional belief $\mu_2(\cdot|\,c_2) \in \Delta(C_1)$ held by the underdog's type c_2 is first-order stochastically decreasing in $c_2 \in C_2$. Then, the conclusion of Theorem 1 continues to hold true.*

Proof. We first show that all information must be revealed in any perfect Bayesian equilibrium. The key point to note is that, even if the underdog's conditional belief $\mu_1(\cdot|\,c_2) \in \Delta(C_1)$ is weakly decreasing in c_2 in the FOSD sense, the underdog's bid schedule remains weakly decreasing globally, as well as strictly decreasing in the interior. Indeed, this follows immediately by combining Lemma B.2 and Lemma C.1, where the underdog's domain condition holds by Lemma 2. Therefore, the proof of Proposition 1, which exploits only the monotonicity properties of the bid schedules and the monotonicity properties of the best-response mappings, extends without change to this more general setting. Thus, the underdog's side of the contest unravels. For the favorite, correlation now does not matter anymore, i.e., Proposition 2 applies as before. This proves the claim. Next, we show that self-disclosure by all types of both players is indeed a perfect Bayesian equilibrium. Even in the presence of arbitrary correlation, this is so provided we keep the specification of off-equilibrium beliefs used in the proof of Theorem 1. The reason is that, under this specification, type-specific payoffs resulting from self-disclosure or unilateral concealment are expected values of complete-information payoffs. Therefore, the correlation does not affect the interim payoff ranking for the contest stage, and the argument proceeds as before. □

The intuition is as follows. The assumption on conditional beliefs means that weaker types of the underdog are more pessimistic, in the sense that they deem stronger types of the favorite more likely. As a result, the best-response bid schedule of the underdog remains strictly declining in the interior, so that the conclusions of the crucial Proposition 1 continue to hold true. Moreover, once the side of the underdog has unraveled, any ex-ante correlation will be resolved, so that full separation obtains as before via Proposition 2.

For similar reasons, the strong-form disclosure principle holds for general forms of correlation provided that the degree of correlation is small enough. For strongly positively correlated types, however, the situation may complicate. In fact, Proposition 1 may break down, as the following example illustrates.

Player 1	$c_2^1 = 5.5$	$c_2^2 = 6$
$c_1^1 = 1$	0.45	0.05
$c_1^2 = 1.5$	0.05	0.45

Player 1		Player 2	
$\xi_1^*(c_1^1) = 0.1318$	$\xi_1^*(c_1^2) = 0.1056$	$\xi_2^*(c_2^1) = 0.0242$	$\xi_2^*(c_2^2) = 0.0262$
$\wedge$	$\wedge$		$\vee$
$\xi_1^\#(c_1^1) = 0.1353$	$\xi_1^\#(c_1^2) = 0.1057$	–	$x_2^\# = 0.0260$

Figure D.1 Positive correlation.

Example D.1 (Positive correlation) Consider the contest specified in Figure D.1. Assumption 1 holds in this example. Shown are the equilibrium bids with and without disclosure by the weak type of the underdog. As can be seen, disclosure induces both types of the favorite to bid higher. Hence, the weak type of the underdog does not benefit from disclosing her type.

The logic of the example is that, without disclosure, the strong type of the favorite expects meeting the strong type of the underdog that, as a result of positive correlation, bids lower than the weak type of the underdog. Therefore, with disclosure, the strong type of the favorite raises her bid. In contrast, disclosure does not substantially change the belief of the favorite's weak type, but she expects meeting the weak type with somewhat higher probability, which makes her bid higher.

Despite the fact that Proposition 1 does not hold literally with positive correlation, Theorem 1 is robust if the type distribution is generic, as has been explained in the body of the paper.

D.2 Noisy signals

It is also of interest to study players' incentives to release noisy signals (not to be confused with randomized revelations). That question is, in general, harder to address. The following result shows for a special case that the weakest type of the underdog, when active, has always a strict incentive to send a noisy signal that corresponds to a first-order increase over her type space, provided that her own type will appear more likely.

Proposition D.2 (Noisy signals) *Consider an unbiased lottery contest, and assume that the type $c_1^{\#}$ of player 1 is public, while the type of player 2 is private information. Suppose that $\underline{c}_2 > c_1^{\#}$, and that type $\overline{c}_2$ is active. Then a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ strictly more likely induces player 1 to strictly lower her effort $x_1^{\#}$.*

Proof. Before the shift, $x_1^{\#,\text{ before}} = E[\sqrt{c_2}]^2/(c_1^{\#} + E[c_2])^2$. It suffices to prove the claim for a FOSD shift in the type distribution of player 2 that makes $\overline{c}_2$ more likely by a probability $\varepsilon > 0$, and another type $\widehat{c}_2 < \overline{c}_2$ less likely by the same probability. Then, after the shift, we get

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})}{c_1^{\#} + E[c_2] + \varepsilon(\overline{c}_2 - \widehat{c}_2)} \right)^2 . \tag{D.1}$$

Let $\widehat{\varepsilon} = \varepsilon(\sqrt{\overline{c}_2} - \sqrt{\widehat{c}_2})$. Then,

$$x_1^{\#,\text{ after}} = \left(\frac{E[\sqrt{c_2}] + \widehat{\varepsilon}}{c_1^{\#} + E[c_2] + \widehat{\varepsilon}(\sqrt{\overline{c}_2} + \sqrt{\widehat{c}_2})} \right)^2 . \tag{D.2}$$

It follows that $x_1^{\#,\text{ after}} < x_1^{\#,\text{ before}}$ holds if and only if

$$c_1^{\#} + E[c_2] < \underbrace{E[\sqrt{c_2 \overline{c}_2}]}_{> E[c_2]} + \underbrace{E[\sqrt{c_2 \widehat{c}_2}]}_{\geq c_1^{\#}} , \tag{D.3}$$

which is always true. The claim follows. □

D.3 Sequential moves

The following result shows that the strong-form disclosure principle in probabilistic contests is robust with respect to sequential disclosures.

Proposition D.4 (Sequential moves) *Suppose that, instead of moving simultaneously at the revelation stage, players move sequentially. Then, imposing Assumption 1, the conclusion of Theorem 1 continuous to hold true under either of the following conditions:*

(i) The favorite moves first;

(ii) the underdog moves first, and the contest is a lottery (i.e., a Tullock contest with $r = 1$).[5]

Proof. (i) If the favorite moves first, then Proposition 1 implies that the Bayesian game beginning with the information set reached by the favorite's decision will unravel on the side of the underdog. Therefore, the most efficient type of the favorite that uses that decision will strictly prefer to reveal her type by Proposition 2. Thus,

[5] We conjecture that the assumption that the contest is a lottery is not needed.

the game unravels on both sides, as claimed. (ii) Suppose next that the underdog moves first. Focus on the weakest type of the underdog, $\bar{c}_2$. If $\bar{c}_2$'s type is revealed (either because she discloses or because all other types disclose), then the favorite will subsequently reveal her type by Proposition 2. Therefore, the contest stage for $\bar{c}_2$ will be of complete information. In contrast, if $\bar{c}_2$ is not revealed, then the contest will feature incomplete information on the side of the underdog (and possibly on the side of the favorite as well). We, therefore, have to show that $E_{c_1}[\Pi_2^{\circ}(c_1,\bar{c}_2)] > E_{c_1}[\Pi_2^*(\xi_2^*(\bar{c}_2),\xi_1^*(c_1))]$. By Proposition 1, $E_{c_1}[\Pi_2(x_2^{\#},\xi_1^{\#}(c_1);\bar{c}_2)] > E_{c_1}[\Pi_2^*(\xi_2^*(\bar{c}_2),\xi_1^*(c_1))]$, so that it suffices to prove $E_{c_1}[\Pi_2^{\circ}(c_1,\bar{c}_2)] \geq E_{c_1}[\Pi_2(x_2^{\#},\xi_1^{\#}(c_1);\bar{c}_2)]$, or equivalently, that

$$E_{c_1}\left[\left(\frac{c_1}{c_1+\bar{c}_2}\right)^2\right] \geq \frac{E[\sqrt{c_1}]^2 E[c_1]}{(E[c_1]+\bar{c}_2)^2}. \tag{D.4}$$

Noting that this inequality is homogeneous of degree zero, we may assume without loss of generality that $\bar{c}_2 = 1$. But the resulting inequality,

$$E_{c_1}\left[\left(\frac{c_1}{c_1+1}\right)^2\right] \geq \frac{E[\sqrt{c_1}]^2 E[c_1]}{(E[c_1]+1)^2}, \tag{D.5}$$

corresponds to the conclusion of Lemma G.1(ii) with $g(x,y) = x^2y/(y+1)^2$. It is straightforward to verify that

$$(d_x\ d_y)\,(H_g(x,y))\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2yd_x^2}{(y+1)^2}\left(1+\frac{x(2-y)}{y\,(y+1)}\frac{d_y}{d_x}\right)\left(1-\frac{x}{y+1}\frac{d_y}{d_x}\right). \tag{D.6}$$

It therefore suffices to show that

$$\left\{x,y\in(0,1), y\geq x^2, d_x>0, d_y>0, \frac{d_y}{d_x}<\frac{1-y}{1-x}\right\} \Rightarrow \left\{\frac{x}{y+1}\frac{d_y}{d_x}<1\right\}. \tag{D.7}$$

The conclusion follows if $\frac{1-y}{1-x} \leq \frac{y+1}{x}$, which is easily seen to be equivalent to $2x \leq 1+y$, which in turn holds true because $y \geq x^2$. $\square$

D.4 Proof of Theorem 2

Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \bar{c}_2\}$ is private with $K_2 \geq 2$. By a *marginal piece of evidence*, we mean a K_2-dimensional vector $\delta_2 = (\delta_2^1, \ldots, \delta_2^{K_2})$ such that $\sum_{k=1}^{K_2} \delta_2^k = 0$. The intuition is that δ_2 turns i's prior belief $q_2 \in \Delta(C_2)$ about player 2's type into a nearby posterior $\widetilde{q}_2 \in \Delta(C_2)$ such that $\widetilde{q}_2(c_2^k) = q_2(c_2^k) + \varepsilon\delta_2^k$, where $\varepsilon > 0$ is a small positive number. Given our assumption that all types have a positive ex-ante probability, adding a marginal piece of evidence for small enough $\varepsilon > 0$ will always be feasible in a comparative statics exercise.

Lemma D.1 (Necessary and sufficient conditions) *Suppose that player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in C_2 = \{\underline{c}_2 \equiv c_2^1, \ldots, c_2^{K_2} \equiv \bar{c}_2\}$ is private with $K_2 \geq 2$. Suppose also that all types of player 2 are active. Then, there exists a positive and strictly hump-shaped (which includes the possibility of strictly monotone increasing or strictly monotone decreasing) sequence $(\varphi^1, \ldots, \varphi^{K_2})$ such that, for any marginal piece of evidence δ_2, any type $c_2 \in C_2$ strictly prefers disclosing δ_2 over concealing δ_2 if and only if*

$$\begin{pmatrix} \delta_2^1 \\ \vdots \\ \delta_2^{K_2} \end{pmatrix} \cdot \begin{pmatrix} \varphi^1 \\ \vdots \\ \varphi^{K_2} \end{pmatrix} < 0. \tag{D.8}$$

Proof. We use the shorthand notation $x_2^k \equiv \xi_2^{\#}(c_2^k)$ and $p_2^k \equiv p_2(x_2^k, x_1^{\#})$ for $k \in \{1, \ldots, K_2\}$. We have the first-order conditions

$$\sum_{k=1}^{K_2} \widetilde{q}_2^k p_2^k(1-p_2^k) = c_1^{\#}\Phi(x_1^{\#}), \tag{D.9}$$

$$p_2^k(1-p_2^k) = c_2^k\Phi(x_2^k) \qquad (k \in \{1,\ldots,K_2\}). \tag{D.10}$$

Totally differentiating (D.9) and evaluating at $\varepsilon = 0$ yields

$$\sum_{k=1}^{K_2} q_2^k(1-2p_2^k)dp_2^k + \left\{\sum_{k=1}^{K_2} \delta_2^k p_2^k(1-p_2^k)\right\} d\varepsilon = c_1^{\#}\Phi'(x_1^{\#})dx_1^{\#}. \tag{D.11}$$

Moreover, for $k \in \{1,\ldots,K_2\}$, using (D.10),

$$\begin{aligned} dp_2^k &= \frac{\partial p_2^k}{\partial x_1^{\#}}dx_1^{\#} + \frac{\partial p_2^k}{\partial x_2^k}dx_2^k & \text{(D.12)}\\ &= -\frac{p_2^k(1-p_2^k)}{\Phi(x_1^{\#})}dx_1^{\#} + \frac{p_2^k(1-p_2^k)}{\Phi(x_2^k)}dx_2^k & \text{(D.13)}\\ &= c_2^k\left\{-\frac{\Phi(x_2^k)}{\Phi(x_1^{\#})}dx_1^{\#} + dx_2^k\right\}. & \text{(D.14)} \end{aligned}$$

From Lemma B.1(iv),

$$dx_2^k = \frac{\Phi(x_2^k)}{\Phi(x_1^{\#})}\frac{2p_2^k-1}{\Phi'(x_2^k)-1+2p_2^k}dx_1^{\#}, \tag{D.15}$$

so that

$$dp_2^k = -c_2^k\frac{\Phi(x_2^k)}{\Phi(x_1^{\#})}\frac{\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}dx_1^{\#} \qquad (k \in \{1,\ldots,K_2\}). \tag{D.16}$$

Using (D.16) to eliminate dp_2^k in (D.11), we obtain

$$\begin{aligned} \left\{\sum_{k=1}^{K_2}\delta_2^k c_2^k\Phi(x_2^k)\right\}d\varepsilon &= \frac{dx_1^{\#}}{\Phi(x_1^{\#})}\left\{c_1^{\#}\Phi(x_1^{\#})\Phi'(x_1^{\#}) + \sum_{k=1}^{K_2} q_2^k c_2^k\Phi(x_2^k)\frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}\right\} & \text{(D.17)}\\ &= \frac{dx_1^{\#}}{\Phi(x_1^{\#})}\sum_{k=1}^{K_2} q_2^k c_2^k\Phi(x_2^k)\left\{\Phi'(x_1^{\#}) + \frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)-1+2p_2^k}\right\}. & \text{(D.18)} \end{aligned}$$

We claim that

$$\Phi'(x_1^{\#}) + \frac{(1-2p_2^k)\Phi'(x_2^k)}{\Phi'(x_2^k)+2p_2^k-1} > 0 \qquad (k \in \{1,\ldots,K_2\}). \tag{D.19}$$

Indeed, this is obvious for $p_2^k \leq \frac{1}{2}$ since $\Phi' \geq 1$ by Lemma B.1(ii).[6] On the other hand, if $p_2^k > \frac{1}{2}$, then $2p_2^k - 1 > 0$, and hence,

$$(2p_2^k-1)\cdot\frac{\Phi'(x_2^k)}{\Phi'(x_2^k)+2p_2^k-1} < 2p_2^k - 1 \leq 1 \leq \Phi'(x_1^{\#}). \tag{D.20}$$

[6] Note also that $p_2^k > 0$ since $x_2^k > 0$ by assumption.

Hence, claim (D.19) is indeed true. Therefore,

$$\frac{dx_1^\#}{d\varepsilon} = \{\text{sth. positive}\}^{-1} \cdot \left\{ \sum_{k=1}^{K_2} \delta_2^k \varphi^k \right\}, \tag{D.21}$$

where $\varphi^k = c_2^k \Phi(x_2^k)$, for $k \in \{1, \ldots, K_2\}$. The strict hump-shape of the sequence $(\varphi^1, \ldots, \varphi^{K_2})$ follows immediately from (D.10) and Lemma B.2. This proves the lemma. □

This lemma offers a necessary and sufficient condition for letting any type $c_2 \in C_2$ prefer to release a marginal piece of evidence δ_2. As will be noted, the condition does not depend on c_2, which means that all types have the same preference for disclosure. Obviously, this is due to the assumption of one-sided asymmetric information, which implies that a decline of $x_1^\#$ is equally desirable for all types of player 2.[7] Note also that Lemma D.1 does not require that the contest is uniformly asymmetric. However, the interiority assumption is crucial. Indeed, inactive types may not have a strict incentive to disclose a marginal piece of evidence if it does not change their state of marginalization. Thus, marginalized types (with positive shadow costs) exhibit some inertia with respect to the release of a marginal piece of evidence.

Condition (D.8) gets a simple interpretation in the Tullock case, where it turns out that we may choose $\varphi^k = c_2^k \xi_2^\#(c_2^k)$, for $k \in \{1, \ldots, K_2\}$, to be the type-specific *equilibrium costs* (or *expenses*). In an interior equilibrium, these costs indeed exhibit the hump-shape described in Lemma D.1 as a consequence of the first-order condition. Thus, in the Tullock case, a marginal piece of evidence is preferred to be disclosed if, roughly speaking, it makes extremal types (i.e., those with the lowest equilibrium costs) more likely and central types (i.e., those with highest equilibrium costs) less likely. Here as well, the activity assumption is crucial to obtain the conclusion of voluntary disclosure. Note, however, that Lemma D.1 does not admit an unraveling conclusion.

Lemma D.2 *Suppose given generic cost parameters $c_1^\# > 0$ and $\overline{c}_2 > \underline{c}_2 > 0$. Then, at least one of the following two statements holds true:*
(i) $c_1^\# \Phi(x_1^\#) > \underline{c}_2 \Phi(\xi_2^\#(\underline{c}_2))$, for all q_2 that assign positive probability to all types in C_2;
(ii) $c_1^\# \Phi(x_1^\#) > \overline{c}_2 \Phi(\xi_2^\#(\overline{c}_2))$, for all q_2 that assign positive probability to all types in C_2.

Proof. As before, we denote by $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ type c_2's best response to the deterministic bid x_1. Let $e_2(x_1; c_2) = c_2 \Phi(\widetilde{\beta}_2(\psi_1(x_1); c_2))$.[8] The relevance of this function stems from the fact that, in equilibrium, $c_1^\# \Phi(x_1^\#) = E[e_2(x_1^\#; c_2)]$. We claim that the two functions $e_2(x_1; \underline{c}_2)$ and $e_2(x_1; \overline{c}_2)$ have the following *single-crossing property*: There exists some threshold value $\widehat{x}_1$ such that $e_2(x_1; \underline{c}_2) \gtrless e_2(x_1; \overline{c}_2)$ if and only if only if $x_1 \gtrless \widehat{x}_1$ (in the interval where $x_1 > 0$ and $\widetilde{\beta}_2(\psi_1(x_1); \underline{c}_2) > 0$). To see why the single-crossing property holds, recall first that $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ is strictly hump-shaped in x_1. Therefore, given Lemma B.1(ii), the same is true for $e_2(x_1; c_2)$. The maximum of the function $e_2(x_1; c_2)$ is $\frac{1}{4}$, and that maximum is reached at $x_1 = x_1^{\max}(c_2)$ characterized by $p_2(\widetilde{\beta}_2(\psi_1(x_1); c_2), x_1) = \frac{1}{2}$. As $\widetilde{\beta}_2(\psi_1(x_1); c_2)$ is strictly declining in c_2 in the interior by Lemma B.2, we have that $x_1^{\max}(c_2)$ is strictly declining in c_2. Now, there are three cases. First, in the interval $[x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2)]$, the function $e_2(x_1; \overline{c}_2)$ is strictly declining in x_1 when positive, while the function $e_2(x_1; \overline{c}_2)$ is strictly increasing in x_1 when positive. Moreover, both functions are continuous (even differentiable when positive by the implicit function theorem). Hence, there exists a unique $\widehat{x}_1 \in (x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2))$ such that the single-crossing property holds in the interval $[x_1^{\max}(\overline{c}_2), x_1^{\max}(\underline{c}_2)]$. Next, in the interval $(0, x_1^{\max}(\overline{c}_2))$, we claim that $e_2(x_1; \underline{c}_2) < e_2(x_1; \overline{c}_2)$. To see why, note that total differentiation of type c_2's first-order condition yields

$$\frac{de_2}{dc_2} = \frac{(1 - 2p_2)\frac{\partial p_2}{\partial x_2}\Phi}{\text{SOC}} > 0, \tag{D.22}$$

where SOC< 0 stands for the second derivative of type c_2's payoff function and we dropped the arguments for convenience. Note that $p_2 > \frac{1}{2}$ because $x_1 < x_1^{\max}(\overline{c}_2)$. Thus, lowering c_2 gradually from $\overline{c}_2$ down to $\underline{c}_2$, we indeed find that $e_2(x_1; \underline{c}_2) < e_2(x_1; \overline{c}_2)$. Finally, for values $x_1 > x_1^{\max}(\overline{c}_2)$ such that $\widetilde{\beta}_2(\psi_1(x_1); \underline{c}_2) > 0$, one shows

[7] This contrasts with the case of two-sided asymmetric information, further dealt with below, where preferences regarding information release may in general differ across types.

[8] In the case of the lottery contest, $e(x_1, c_2)$ is type c_2's expense when using a best response to x_1.

that $e_2(x_1;\underline{c}_2) > e_2(x_1;\overline{c}_2)$. The argument is essentially the same as before, provided that one notes that $p_2 < \frac{1}{2}$ because $x_1 > x_1^{\max}(\overline{c}_2)$. Now, the intersection point of the two functions $e_2(x_1;\underline{c}_2)$ and $e_2(x_1;\overline{c}_2)$ lies either below, on, or above the function $x_1 \mapsto c_1^{\#}\Phi(x_1)$, regardless of the probability distribution q_2. In the first case ("below"), we know from single-crossing that $e_2(x_1;\underline{c}_2) < e_2(x_1;\overline{c}_2)$. Hence, we are in case (i). In the second case ("on"), we know that $e_2(x_1;\underline{c}_2) = e_2(x_1;\overline{c}_2)$, which is a non-generic case, and leads to case (i) if $K_2 \geq 3$. In the third case ("above"), we are in case (ii). The lemma follows. □

Proof of Theorem 2. A type that would be inactive at the contest stage always strictly prefers to reveal her private information. Therefore, one may assume without loss of generality that the equilibrium at the contest stage is interior. But then, the vector $\{c_2^k\Phi(x_2^k)\}_{k=1}^{K_2}$ is strictly hump-shaped as a consequence of the first-order condition $p_2^k(1-p_2^k) = c_2^k\Phi(x_2^k)$, for any $k \in \{1,\ldots,K_2\}$, and the strict declining monotonicity of the bid schedule $\{x_2^k\}_{k=1}^{K_2}$. Now, the "all-or-nothing" disclosure by a type $c_2^k \in C_2$ may be seen as a continuous accumulation of identical pieces of evidence δ_2 with $\delta_2(c_2^k) = 1 - q_2^k$ and $\delta_2(c_2^l) = -q_2^l$ for any $l \in \{1,\ldots,K_2\}$ such that $l \neq k$. Suppose first that $K_2 \geq 3$. Then, by Lemma D.2, there exists an extremal type $c_2^k \in \{\underline{c}_2, \overline{c}_2\}$ such that, for the just defined marginal piece of evidence, the condition in Lemma D.1 is fulfilled. Moreover, this condition remains valid on the entire path. Hence, there is one extremal type that strictly prefers to disclose. Suppose next that $K_2 = 2$. Then the same argument goes through for generic values of $c_1^{\#}$, $\underline{c}_2$, and $\overline{c}_2$. This proves the claim. □

D.5 Continuous types

As a solution concept for the contest stage, we use pure-strategy Nash equilibrium rather than Bayesian equilibrium. The reduced-form definition of perfect Bayesian equilibrium remains unchanged. However, to ensure continuity properties of type-specific payoffs, we restrict attention to the special case of the lottery contest. A formal statement reflecting our discussion in the body of the paper is the following.

Proposition D.4 (Continuous type distributions) *Consider a uniformly asymmetric lottery contest with continuous and independent type distributions. Then, in any perfect Bayesian equilibrium of the contest with pre-play communication of verifiable information, both contestants' types are almost surely revealed at the contest stage.*

Proof. We repeatedly apply Benoît and Dubra (2006, Thm. 1), for which we refer the reader to the original paper. In a first step, we note that type c_2's expected payoff from disclosure,

$$u_2(c_2,c_2,S_1,S_2) \equiv \int_{S_1} \Pi_2(x_1^{\circ},x_2^{\circ};c_2)dF_1(c_1) + \int_{C_1\backslash S_1} \Pi_2(\xi_1^{\#}(c_1),x_2^{\#};c_2)dF_1(c_1) \tag{D.23}$$

$$= \int_{S_1}\left(\frac{c_1}{c_1+c_2}\right)^2 dF_1(c_1) + \mathrm{pr}\{C_1\backslash S_1\}\frac{E[\sqrt{c_1}|\,c_1\in C_1\backslash S_1]^2 E[c_1|\,c_1\in C_1\backslash S_1]}{\left((E[c_1|\,c_1\in C_1\backslash S_1]+c_2\right)^2}, \tag{D.24}$$

is continuous in c_2 by Lebesgue's theorem of dominated convergence. Similarly, type c_2's expected payoff from concealment,

$$u_2(c_2,\emptyset,S_1,S_2) \equiv \int_{S_1} \Pi_2(x_1^{\#},\xi_2^{\#}(c_2);c_2)dF_1(c_1) + \int_{C_1\backslash S_1} \Pi_2(\xi_1^{*}(c_1),\xi_2^{*}(c_2);c_2)dF_1(c_1) \tag{D.25}$$

$$= \mathrm{pr}\{S_1\}\left(1-\frac{\sqrt{c_2}E[\sqrt{\widetilde{c}_2}|\,\widetilde{c}_2\in C_2\backslash S_2]}{c_1+E[\widetilde{c}_2|\,\widetilde{c}_2\in C_2\backslash S_2]}\right)^2 + \int_{C_1\backslash S_1}\left(\frac{\xi_1^*(c_1)}{\xi_1^*(c_1)+\xi_2^*(c_2)} - c_2\xi_2^*(c_2)\right)dF_1(c_1), \tag{D.26}$$

is well-defined by the existence and uniqueness results in Ewerhart (2014, Thm. 3.4 & 4.2). Moreover, the mapping $c_2 \mapsto u_2(c_2,\emptyset,S_1,S_2)$ is continuous because ξ_1^* in the second term does not depend on c_2. By a straightforward generalization of Proposition 1, for any non-degenerate conditional distribution $F_2(\cdot|\,\emptyset,S_2)$, the lowest type $\widehat{c}_2$ in the support of $F_2(\cdot|\,\emptyset,S_2)$ has the property that $u_2(\widehat{c}_2,\widehat{c}_2,S_1,S_2) > u_2(\widehat{c}_2,\emptyset,S_1,S_2)$. By Benoît and Dubra (2006, Thm. 1), the underdog's signal is almost surely known in any perfect Bayesian equilibrium. Next, we consider the decision of the favorite under the assumption that the underdog's type is revealed with probability one. Continuity of the expected payoff functions $u_1(c_1,c_1,S_1,C_2)$ and $u_1(c_1,\emptyset,S_1,C_2)$, defined in analogy to (D.23) and (D.25), may be checked as above. Then, by a straightforward generalization of Proposition 2, almost surely across c_2 and for any non-degenerate conditional distribution $F_1(\cdot|\,\emptyset,S_1)$, the highest type $\widehat{c}_1$ in the support of $F_1(\cdot|\,\emptyset,S_1)$ has typewise across $c_2 \in C_2$, but hence also globally, the property that $u_1(\widehat{c}_1, \widehat{c}_1, S_1, C_2) > u_1(\widehat{c}_1, \emptyset, S_1, C_2)$. Applying Benoît and Dubra (2006, Thm. 1) again, we see that also the favorite's type is necessarily almost surely known in any perfect Bayesian equilibrium. This concludes the proof and proves the proposition. □

D.6 Other types of uncertainty

Proposition 3 says that our focus on marginal cost types is essentially without loss of generality. The proof relies on suitable variable substitutions.

Proof of Proposition 3. (i) Suppose first that the ability parameters γ_1 and γ_2 are public information. Then, using the substitution $\widetilde{c}_i = c_i/(V_i - L_i)$, the positive affine transform of type θ_i's payoff function,

$$\frac{\Pi_i(x_i, x_j; \theta_i) - L_i}{V_i - L_i} = \frac{\gamma_i h(x_i)}{\gamma_1 h(x_1) + \gamma_2 h(x_2)} - \frac{c_i}{V_i - L_i} x_i = \Pi_i(x_i, x_j; \widetilde{c}_i), \tag{D.27}$$

is seen to be of the normalized type assumed above. (ii) Suppose, alternatively, that $h(y) = y^r$ for some $r \in (0, 1]$. Then, using the substitution $\widetilde{x}_i = \gamma_i^{-1/r} x_i$, one finds similarly that

$$\frac{\Pi_i(x_i, x_j; \theta_i) - L_i}{V_i - L_i} = \frac{(\gamma_i^{1/r} x_i)^r}{(\gamma_1^{1/r} x_1)^r + (\gamma_2^{1/r} x_2)^r} - \frac{c_i}{V_i - L_i}(\gamma_i^{-1/r} x_i) = \Pi_i(\widetilde{x}_i, \widetilde{x}_j; \widetilde{c}_i). \tag{D.28}$$

This completes the proof. □

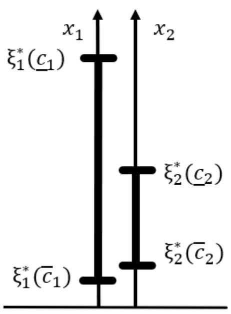
E. Material omitted from Section 6

Section 6 is concerned with limits of the scope of the strong-form disclosure principle. We will discuss contests that are not uniformly asymmetric, Bayesian persuasion, the option to shut down communication, and unverifiable information.

E.1 Contests that are not uniformly asymmetric

In Example 1, bid distributions under full concealment are overlapping. A similar example in which bid distributions are nested is shown below.

Example E.1 (Nested type distributions) The contest specified by the parameters shown in Figure E.1 is not uniformly asymmetric. Assumption 1 does not hold. And indeed, as table (b) in the figure shows, there is a perfect Bayesian equilibrium in which no type reveals her private information.



Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 0.7$	$c_1^2 = 7$ $q_1(c_1^2) = 0.3$	$c_2^1 = 2$ $q_2(c_2^1) = 0.8$	$c_2^2 = 5$ $q_2(c_2^2) = 0.2$
$\xi_1^*(c_1^1) = 0.2016$ $\Pi_1^*(c_1^1) = 0.5031$	$\xi_1^*(c_1^2) = 0.0253$ $\Pi_1^*(c_1^2) = 0.0718$	$\xi_2^*(c_2^1) = 0.1019$ $\Pi_2^*(c_2^1) = 0.2717$	$\xi_2^*(c_2^2) = 0.0310$ $\Pi_2^*(c_2^2) = 0.1036$
$x_1^\# = 0.1923$ $\Pi_1^\# = 0.4999$	–	$\xi_2^\#(c_2^1) = 0.1178$	$\xi_2^\#(c_2^2) = 0.0038$
–	$x_1^\# = 0.0270$ $\Pi_1^\# = 0.0703$	$\xi_2^\#(c_2^1) = 0.1178$	$\xi_2^\#(c_2^2) = 0.0038$
$\xi_1^\#(c_1^1) = 0.2144$	$\xi_1^\#(c_1^2) = 0.0208$	$x_2^\# = 0.0968$ $\Pi_2^\# = 0.2712$	–
$\xi_1^\#(c_1^1) = 0.1548$	$\xi_1^\#(c_1^2) = 0.0357$	–	$x_2^\# = 0.0367$ $\Pi_2^\# = 0.1027$

(a) (b)

Figure E.1 Nested type distributions.

One might conjecture that the strong-form disclosure principle *generally* fails in symmetric contests of incomplete information. The following example shows that this is not the case.

Example E.2 (Symmetric contests) Consider a symmetric lottery contest with two equally likely types, $\overline{c} > \underline{c} > 0$, for each player (Malueg and Yates, 2004). This contest is not uniformly asymmetric.[9] Theorem

[9] It suffices to note that, by Lemma 1, the Nash equilibrium after full disclosure is unique, hence symmetric and interior if two equal types are matched.

2 implies that one-sided disclosure cannot be an equilibrium. To derive the conditions under which mutual concealment is an equilibrium, suppose that player 2 does not disclose her private information. Then, the efficient type of player 1, $\underline{c}$, has expected payoffs from not disclosing of

$$E_{c_2}[\Pi_1(\xi_1^*(\underline{c}), \xi_2^{\#}(c_2)); \underline{c}] = \frac{1}{8} + \frac{\overline{c}^2}{2(\underline{c} + \overline{c})^2}. \tag{E.1}$$

If $\underline{c}$, instead, reveals her private information, then there are two cases. If $\overline{c}/\underline{c} \geq 9$, disclosure marginalizes the inefficient type of player 2, i.e, $\xi_2^{\#}(\underline{c}, \overline{c}) = 0$. As a result, two equally efficient opponents meet with probability one half, so that player 1's expected payoff is $E_{c_2}[\Pi_1(x_1^{\#}(\underline{c}), \xi_2^{\#}(c_2; \underline{c})); \underline{c}] = \frac{5}{9}$. Therefore, revealing her type is optimal for the efficient type if $\overline{c}/\underline{c} < \frac{6\sqrt{31}+31}{5} \approx 12.88$. For the inefficient type of player 1, $\overline{c}$, one compares the expected payoff from not disclosing,

$$E_{c_2}[\Pi_1(\xi_1^*(\overline{c}), \xi_2^*(c_2)); \overline{c}] = \frac{1}{8} + \frac{\underline{c}^2}{2(\underline{c} + \overline{c})^2}, \tag{E.2}$$

with the expected payoff in the contest with one-sided private information,

$$E_{c_2}[\Pi_1(x_1^{\#}(\underline{c}), \xi_2^{\#}(c_2; \underline{c})); \underline{c}] = \frac{(\sqrt{\underline{c}} + \sqrt{\overline{c}})^2(\underline{c} + \overline{c})}{2(\underline{c} + 3\overline{c})^2}, \tag{E.3}$$

which can be easily checked to be always strictly lower. Thus, the strong-form disclosure principle holds in this example if and only if $\overline{c}/\underline{c} < 12.88$.[10]

E.2 Bayesian persuasion

Kamenica and Gentzkow (2011) considered a general setting with one sender and one receiver, and an unknown state of the world, where the sender precommits to a signal about the state of the world. Upon receiving the signal, the receiver rationally updates her belief about the state of the world and takes an action. Depending on whether the commitment power lies with a player or with the social planner, the approach is known as Bayesian persuasion or information design. In this section, we consider the first problem, i.e., the sender is a player.[11]

Consider a lottery contest with one-sided incomplete information, where the type $c_1^{\#}$ of player 1 is public. Then the expected payoff to an active type c_2 is given by

$$\Pi_2^{\#}(c_2|\mu_2) = \left(1 - \frac{\sqrt{c_2} E[\mathbf{1}_{\widetilde{c}_2 \text{ active}} \sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\mathbf{1}_{\widetilde{c}_2 \text{ active}} \widetilde{c}_2|\mu_2]}\right)^2, \tag{E.4}$$

where $\mathbf{1}_{\widetilde{c}_2 \text{ active}}$ is an indicator variable that equals one (zero) if $\xi_2^{\#}(\widetilde{c}_2; c_1^{\#}) > 0$ $(= 0)$ in the contest with one-sided incomplete information and type distribution μ_2 for player 2, and where $E[.|\mu_2]$ denotes the expectation given player 1's belief $\mu_2 \in \Delta(C_2)$ about 2's types at the contest stage.[12] The logic of marginalization is as follows. As the belief μ_2 gives too much weight to strong types, player 1 bids higher which induces weak types of player 2 to bid zero. The condition spelt out in the following lemma ensures that marginalization does not occur for a persuasion model with two types.

[10] This example helps clarifying the relationship to the corresponding analysis of the all-pay auction. Kovenock et al. (2015, Prop. 5) found that, with either private or common values, the interim information sharing game admits a perfect Bayesian equilibrium in which no firm ever shares its information. As noted by a referee, there is an important caveat here because the literature on all-pay auctions (Kovenock et al., 2015; Tan, 2016) has tended to focus on symmetric contests, while our analysis has focused on asymmetric contests. However, Example E.2 suggests that it is also the contest technology that matters. Indeed, the difference in conclusions might mirror a more general fact. While the auction induces a hide-and-seek type of randomized behavior for which keeping secrets seems advisable, the probabilistic contest induces players to think in trade-offs, which may then entail voluntary disclosures of private information to the opponent. More work on the relationship of these two "battle modes" in contests is certainly desirable.

[11] The problem of information design is discussed in Subsection F.3.

[12] The expected payoff of an inactive type is zero.

Lemma E.1 (Interiority condition) *Suppose that* $K_2 = 2$ *and* $c_1^{\#} + \underline{c}_2 > \sqrt{\underline{c}_2 \overline{c}_2}$*. Then, all types* $c_2 \in C_2$ *of player 2 are active, regardless of player 1's posterior belief* μ_2*.*

Proof. Take a posterior belief $\mu_2 \in \Delta(C_2)$. The weakest type $\overline{c}_2 \in C_2$ is active if and only if $\sqrt{\overline{c}_2} < (c_1^{\#} + E[c_2|\,\mu_2])/E[\sqrt{c_2}|\,\mu_2]$, or equivalently, if $m\sqrt{\underline{c}_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2}) < c_1^{\#}$, where $m = \mu_2(\underline{c}_2)$. Letting $m = 1$ yields the condition in the statement of the lemma. □

Now, in the absence of communication, μ_2 simply corresponds to the ex-ante distribution $\{q_2(c_2^k)\}_{k=1}^{K_2}$. Bayesian persuasion allows player 2 to precommit to a signal, which induces a probability distribution $\tau_2 \in \Delta(\Delta(C_2))$ over posterior beliefs $\mu_2 \in \Delta(C_2)$ that is subject to Bayes plausibility

$$\int \mu_2(c_2) d\tau_2(\mu_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{E.5}$$

Therefore, player 2's problem reads

$$\max_{\tau_2 \text{ s.t. (E.5)}} \int E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2)\right] d\tau_2(\mu_2), \tag{E.6}$$

where $E_{c_2}[.]$ denotes, as before, the expectation with respect to the prior distribution on C_2 given by q_2.

As a general solution of problem (E.6) is beyond the scope of the present analysis, we discuss a simple example with $K_2 = 2$. Then, with precommitment, the signal may lead to a probability distribution τ_2 over two distributions $\mu_2^{\mathrm{A}}, \mu_2^{\mathrm{B}} \in \Delta(C_2)$, with respective probabilities τ_2^A and τ_2^B satisfying

$$\tau_2^{\mathrm{A}} \mu_2^{\mathrm{A}}(c_2) + \tau_2^{\mathrm{B}} \mu_2^{\mathrm{B}}(c_2) = q_2(c_2) \qquad (c_2 \in C_2). \tag{E.7}$$

For instance, in the special case where $\overline{c}_2 > c_1^{\#} > \underline{c}_2$, we might expect that player 2 benefits if, compared to the prior, μ_2^{A} is biased towards $\underline{c}_2$, while μ_2^{B} is biased towards $\overline{c}_2$. Intuitively, the positive effect of overstatement on the weak type's payoff would be combined with the likewise positive effect of understatement on the strong type's payoff.

Proposition E.1 (Bayesian persuasion) *Consider an unbiased lottery contest where player 1's type* $c_1^{\#}$ *is public, while player 2's type* $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ *is private with* $\overline{c}_2 > \underline{c}_2$*. Suppose also that the interiority assumption of Lemma E.1 holds. Then there exists a threshold value* $\chi^* \in [0, 1]$ *such that:*

(i) If $c_1^{\#} > \sqrt{\underline{c}_2 \overline{c}_2}$*, then full disclosure is optimal;*

(ii) if $c_1^{\#} = \sqrt{\underline{c}_2 \overline{c}_2}$*, then any signal is optimal;*

(iii) if $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$ *and* $q_2(\underline{c}_2) \leq \chi^*$*, then full concealment is optimal;*

(iv) if $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$ *and* $q_2(\underline{c}_2) > \chi^*$*, then player 2 uses a randomized signal with posterior beliefs satisfying* $\mu_2^{\mathrm{A}}(\underline{c}_2) = \chi^*$ *and* $\mu_2^{\mathrm{B}}(\underline{c}_2) = 1$*.*[13]

Proof. The sender (player 2) solves the problem

$$\max_{\tau_2 \text{ s.t. (E.7)}} \tau_2^{\mathrm{A}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}})\right] + \tau_2^{\mathrm{B}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}})\right]. \tag{E.8}$$

More explicitly, this becomes

$$\max_{\substack{\tau_2^{\mathrm{A}} \equiv 1-\tau_2^{\mathrm{B}} \in [0,1], \\ \mu_2^{\mathrm{A}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{A}}(\overline{c}_2) \in [0,1], \\ \mu_2^{\mathrm{B}}(\underline{c}_2) \equiv 1-\mu_2^{\mathrm{B}}(\overline{c}_2) \in [0,1], \\ \text{s.t. (E.7)}}} \tau_2^{\mathrm{A}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{A}})\right] + \tau_2^{\mathrm{B}} E_{c_2}\left[\Pi_2^{\#}(c_2|\mu_2^{\mathrm{B}})\right]. \tag{E.9}$$

[13] An example illustrating this case is discussed below.

We shart with the case where $c_1^{\#} \geq \sqrt{\underline{c}_2 \overline{c}_2}$. By Lemma E.1, both types of player 2 are active. Therefore, the question if player 2 benefits from persuasion (or not) is linked to the strict convexity (or weak concavity) of the function

$$\widehat{\Pi}_2(\mu_2) = q_2(\underline{c}_2) \left(1 - \frac{\sqrt{\underline{c}_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2 + q_2(\overline{c}_2) \left(1 - \frac{\sqrt{\overline{c}_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2, \tag{E.10}$$

where the posterior μ_2 is given as

$$\mu_2 \equiv (\mu_2(\underline{c}_2), \mu_2(\overline{c}_2)) \in \Delta(C_2) = \{(m, 1-m) : 0 \leq m \leq 1\}. \tag{E.11}$$

Let $c_2 \in C_2$. Based on the computation of the first derivative,

$$\frac{\partial}{\partial m} \left(1 - \frac{\sqrt{c_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2 = \frac{\partial}{\partial m} \left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2} \right)^2 \tag{E.12}$$

$$= 2 \cdot \left(1 - \frac{\sqrt{c_2}(m\sqrt{\underline{c}_2} + (1-m)\sqrt{\overline{c}_2})}{c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2} \right) \cdot \frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^2}, \tag{E.13}$$

we see that the second derivative is given by

$$\frac{\partial^2}{\partial m^2} \left(1 - \frac{\sqrt{c_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)^2 = 2 \cdot \left(\frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^2} \right)^2 \tag{E.14}$$

$$+ 4 \cdot \underbrace{\left(1 - \frac{\sqrt{c_2} E[\sqrt{\widetilde{c}_2}|\mu_2]}{c_1^{\#} + E[\widetilde{c}_2|\mu_2]} \right)}_{>0 \text{ by activity}} \cdot \frac{\sqrt{c_2}(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})(c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})(\overline{c}_2 - \underline{c}_2)}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^3}.$$

Clearly, the right-hand side of (E.14) is positive (zero) if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$) regardless of $c_2 \in C_2$ and $m \in [0,1]$, which proves parts (i) and (ii). Suppose next that $\sqrt{\underline{c}_2\overline{c}_2} > c_1^{\#}$. Then, combining (E.10) and (E.14), we get

$$\frac{\partial^2 \widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})^2 (c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + E[\widetilde{c}_2|\mu_2])^4} \cdot \left\{ \begin{array}{c} E[c_2](c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2}) \\ +2(c_1^{\#} + E[\widetilde{c}_2|\mu_2]) E[\sqrt{c_2}](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) \\ -2E[c_2] E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) \end{array} \right\}. \tag{E.15}$$

Exploiting that $E[\sqrt{c_2}](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) = E[c_2] + \sqrt{\underline{c}_2\overline{c}_2}$ and $E[\sqrt{\widetilde{c}_2}|\mu_2](\sqrt{\overline{c}_2} + \sqrt{\underline{c}_2}) = E[\widetilde{c}_2|\mu_2] + \sqrt{\underline{c}_2\overline{c}_2}$, we see that

$$\frac{\partial^2 \widehat{\Pi}_2(\mu_2)}{\partial m^2} = \frac{2(\sqrt{\overline{c}_2} - \sqrt{\underline{c}_2})^2 (c_1^{\#} - \sqrt{\underline{c}_2\overline{c}_2})}{(c_1^{\#} + m\underline{c}_2 + (1-m)\overline{c}_2)^4} \cdot \left\{ \begin{array}{c} (3E[c_2] + 2\sqrt{\underline{c}_2\overline{c}_2})\, c_1^{\#} \\ -\left(3E[c_2] - 2(m\underline{c}_2 + (1-m)\overline{c}_2)\right) \sqrt{\underline{c}_2\overline{c}_2} \end{array} \right\}. \tag{E.16}$$

As the expression in the curly brackets is linear in m, we certainly find a unique cut-off level $m^* \in \mathbb{R}$ such that, if replaced for m in (E.16), renders this term equal to zero. Moreover, $\widehat{\Pi}_2(\mu_2)$ is strictly concave for $m \leq m^*$, and strictly convex for $m \geq m^*$. There are now three cases. Suppose first that $m^* \geq 1$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly concave regardless of $q_2(\underline{c}_2)$, so that full concealment is optimal. In this case, we may set $\chi^* = 1$. Next, suppose that $m^* \in (0,1)$. Then, taking the convex closure of $\widehat{\Pi}_2(\mu_2)$ over the interval $[0,1]$, we find a "tangential" point at some $\chi^* \in [0, m^*)$, as illustrated conceptually in Figure E.2 in the case where $m^* \in (0,1)$ and $\chi^* > 0$. (For $\chi^* = 0$, the slope of $\widehat{\Pi}_2(\mu_2)$ at $m = 0$ and the slope of the upper boundary of the convex closure may differ). If $q_2(\underline{c}_2) \leq \chi^*$, then full concealment remains optimal. If $q_2(\underline{c}_2) > \chi^*$, however, player 2's signal randomizes, in response to her type and the randomizing commitment device, between the two signals causing Bayesian posteriors μ_2^{A} with $\mu_2^{\mathrm{A}}(\underline{c}_2) = \chi^*$ and μ_2^{B} with $\mu_2^{\mathrm{B}}(\underline{c}_2) = 1$. Suppose, finally, that $m^* \leq 0$. Then, $\widehat{\Pi}_2(\mu_2)$ is globally strictly convex regardless of $q_2(\underline{c}_2)$, so that full disclosure is optimal. In this case, we may set $\chi^* = 0$. This proves the claim. □

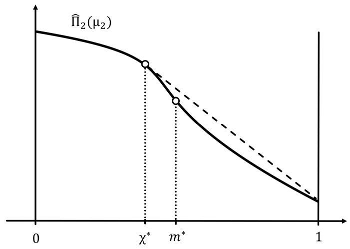


Figure E.2 Bayesian persuasion.

The rough intuition for the underlying effects here is that a stronger uninformed contestant raises her efforts in response to uncertainty, whereas a weaker uninformed contestant lowers her efforts in response to uncertainty (cf. Hurley and Shogren, 1998a). With $c_1^{\#} > \sqrt{\underline{c}_2 \overline{c}_2}$, player 1 is comparably weak, so it makes sense for player 2 to inform player 1. In the knife-edge case where $c_1^{\#} = \sqrt{\underline{c}_2 \overline{c}_2}$, player 1 does not care about player 2's type as each type chooses the same bid level. Hence, any signal is optimal in that case. The situation gets more structured for $c_1^{\#} < \sqrt{\underline{c}_2 \overline{c}_2}$, where player 1 is comparably strong. In that case, the signal will never be fully informative. Instead, either full concealment is optimal (if $q_2(\underline{c}_2) \leq \chi^*$), or player 2 optimally uses a randomized signal (if $q_2(\underline{c}_2) > \chi^*$). When a randomized signal is used, player 2 reveals her type when strong with a probability τ_2^{B} strictly smaller than one, but never reveals her type when weak. As we have shown in the proof of Proposition E.1, player 2's expected payoff $\widehat{\Pi}_2(\mu_2)$ in a contest with posterior μ_2, considered as a function of m, is concave left of some cut-off value m^* and convex right of m^*.

We conclude this subsection by giving an example that illustrates the possibility of a randomizing commitment device.

Example E.3 (Randomization in Bayesian persuasion) Suppose that $c_1^{\#} = 1$, $\underline{c}_2 = 5$, $\overline{c}_2 = 6$, $q_2(\underline{c}_2) = 0.75$, and $q_2(\overline{c}_2) = 0.25$. Then, $m^* = 0.56$ and $\chi^* = 0.32$.

E.3 Shutting down communication

So far, we assumed that, if one player discloses, the other player automatically gets informed, and this is commonly known. But in some situations, it may be possible to publicly commit to ignore any information provided by one's opponent.

Proposition E.2 (The underdog never shuts down communication) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the underdog is public information, whereas the favorite has at least two possible type realizations. Then, the underdog's ex-ante expected payoff is strictly higher under full revelation than under mandatory concealment, i.e.,* $\Pi_2^{\mathrm{FR}} > \Pi_2^{\mathrm{MC}}$.

Proof. The underdog's expected profits under mandatory concealment (i.e., the underdog "closes her eyes") and under full revelation (i.e., the underdog "opens her eyes"), respectively, are easily derived as $\Pi_2^{\mathrm{MC}} = E[\sqrt{c_1}]^2 E[c_1]/(E[c_1] + c_2^{\#})^2$ and $\Pi_1^{\mathrm{FR}} = E[c_1^2/(c_1 + c_2^{\#})^2]$. To compare these expressions, we apply Lemma G.1(ii), in which the support of the random variable Y is assumed to be $(0,1)$, with $Y = \sqrt{c_1/c_2^{\#}}$ and $g(x,y) = g_3(x,y) \equiv \frac{x^2 y}{(1+y)^2}$. It suffices to show that, for any $x, y \in (0,1)$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} < \frac{1-y}{1-x}$, the quadratic form

$$(d_x \; d_y)\,(H_{g_3}(x,y)) \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2{d_x}^2}{(1+y)^2} \left(1 - \frac{x}{1+y} \frac{d_y}{d_x}\right) \left(y + \frac{x(2-y)}{1+y} \frac{d_y}{d_x}\right) \tag{E.17}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1}\cdot\frac{d_y}{d_x}<\underbrace{\frac{x}{y+1}\cdot\frac{1-y}{1-x}}_{\text{decreasing in } y}\leq\frac{3x}{x^2+1}\cdot\frac{1-x^2}{1-x}=\frac{x^2+x}{x^2+1}<1. \tag{E.18}$$

This proves the claim. □

Thus, the underdog would never prefer to publicly announce to ignore any information received. The intuitive force behind this result is that the underdog can better target her effort, so that the ex-ante winning probability increases. An analogous result for the favorite is not true, however. Indeed, Example F.1 below will illustrate the possibility that the favorite may benefit from committing to ignore any information released by the underdog.

E.4 Proof of Theorem 3

Our derivation below draws heavily from Pavlov (2013) who established that communication equilibria and Bayesian equilibria are payoff-equivalent in two-player all-pay auctions.

Proof of Theorem 3. By the existence part of Lemma 1, there is a Bayesian Nash equilibrium of the contest stage, $\xi^*=(\xi_1^*,\xi_2^*)\in X_1\times X_2$. Thus, for any $i\in\{1,2\}$, $c_i\in C_i$, and $x_i\in[0,1]$, we have

$$E_{c_j}[\Pi_i(\xi_i^*(c_i),\xi_j^*(c_j);c_i)]\geq E_{c_j}[\Pi_i(x_i,\xi_j^*(c_j);c_i)], \tag{E.19}$$

where $j\neq i$. Suppose given a *communication equilibrium*, consisting of a nonempty, finite set of reports R_i as well as a nonempty, finite set of messages M_i for each player $i\in\{1,2\}$, a coordination mechanism $\pi:R_1\times R_2\to\Delta(M_1\times M_2)$, and (possibly randomized) functions $\rho_i:C_i\to R_i$, $\zeta_i:M_i\times C_i\to[0,1]$, for each player $i\in\{1,2\}$, such that, for all $i\in\{1,2\}$, $c_i\in C_i$, $\widehat{\rho}_i$, and $\widehat{\zeta}_i$,

$$\begin{aligned}&E_{c_j}\left[\sum_{m_1,m_2}\Pi_i(\zeta_i(m_i,c_i),\zeta_j(m_j,c_j);c_i)\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(c_j))\right]\\&\geq E_{c_j}\left[\sum_{m_1,m_2}\Pi_i(\widehat{\zeta}_i(m_i,c_i),\zeta_j(m_j,c_j);c_i)\pi(m_i,m_j|\,\widehat{\rho}_i(c_i),\rho_j(c_j))\right].\end{aligned} \tag{E.20}$$

In particular, inequality (E.20) holds if the deviation $(\widehat{\rho}_i,\widehat{\zeta}_i)$ is given by an uninformative $\widehat{\rho}_i(c_i)\equiv r_i$ (always send the same report $r_i\in R_i$, regardless of type), and $\widehat{\zeta}_i(m_i,c_i)\equiv\xi_i^*(c_i)$.[14] Thus, for all $i\in\{1,2\}$ and $c_i\in C_i$,

$$\begin{aligned}&E_{c_j}\left[\sum_{m_1,m_2}\Pi_i(\zeta_i(m_i,c_i),\zeta_j(m_j,c_j);c_i)\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(c_j))\right]\\&\geq E_{c_j}\left[\sum_{m_1,m_2}\Pi_i(\xi_i^*(c_i),\zeta_j(m_j,c_j);c_i)\pi(m_i,m_j|\,r_i,\rho_j(c_j))\right].\end{aligned} \tag{E.21}$$

Next, replacing x_i by $\zeta_i(m_i,c_i)$ in (E.19), we have for any $i\in\{1,2\}$, $c_i\in C_i$, and $m_i\in M_i$,

$$E_{c_j}[\Pi_i(\xi_i^*(c_i),\xi_j^*(c_j);c_i)]\geq E_{c_j}[\Pi_i(\zeta_i(m_i,c_i),\xi_j^*(c_j);c_i)]. \tag{E.22}$$

Multiplying by $\pi(m_i,m_j|\,\rho_i(c_i),r_j)$, for some $r_j\in R_j$, and summing over all pairs $(m_1,m_2)\in M_1\times M_2$ yields,

[14] Farrell (1985) and Pavlov (2013) assumed that the babbling deviation has all types randomize over reports. As will become clear, our strategy of proof, where all types send the same report, captures the same intuition.

for any $i \in \{1,2\}$ and $c_i \in C_i$,

$$E_{c_j}[\Pi_i(\xi_i^*(c_i), \xi_j^*(c_j); c_i)] \geq E_{c_j}\left[\sum_{m_1,m_2} \Pi_i(\zeta_i(m_i,c_i), \xi_j^*(c_j); c_i)\pi(m_i, m_j|\, \rho_i(c_i), r_j)\right]. \tag{E.23}$$

In (E.21), we replace $r_i = \rho_i(\widetilde{c}_i)$ and take the expectation over $\widetilde{c}_i \in C_i$. Relationship (E.23) is dealt with in an analogous fashion. Adding the two resulting inequalities, noting that the cost terms cancel out, and taking expectations with respect to c_i, delivers

$$\begin{aligned} &E\left[p_i(\xi_i^*(c_i), \xi_j^*(c_j)) + \sum_{m_1,m_2} p_i(\zeta_i(m_i,c_i), \zeta_j(m_j,c_j))\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(c_j))\right] \\ &\geq E\left[\begin{array}{c} \sum_{m_1,m_2} p_i(\xi_i^*(c_i), \zeta_j(m_j,c_j))\pi(m_i,m_j|\,\rho_i(\widetilde{c}_i),\rho_j(c_j)) \\ +\sum_{m_1,m_2} p_i(\zeta_i(m_i,c_i), \xi_j^*(c_j))\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(\widetilde{c}_j)) \end{array}\right], \end{aligned} \tag{E.24}$$

for any $i \in \{1,2\}$. Adding over players, one arrives at

$$\begin{aligned} &\underbrace{E\left[p_i(\xi_i^*(c_i), \xi_j^*(c_j)) + p_j(\xi_j^*(c_i), \xi_i^*(c_j))\right]}_{=1} \\ &+\underbrace{E\left[\sum_{m_1,m_2}\big(p_i(\zeta_i(m_i,c_i),\zeta_j(m_j,c_j)) + p_j(\zeta_j(m_j,c_j),\zeta_i(m_i,c_i))\big)\,\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(c_j))\right]}_{=1} \\ &\geq \underbrace{E\left[\sum_{m_1,m_2}\big(p_i(\xi_i^*(c_i),\zeta_j(m_j,c_j)) + p_j(\zeta_j(m_j,c_j),\xi_i^*(c_i))\big)\,\pi(m_i,m_j|\,\rho_i(\widetilde{c}_i),\rho_j(c_j))\right]}_{=1} \\ &+\underbrace{E\left[\sum_{m_1,m_2}\big(p_i(\zeta_i(m_i,c_i),\xi_j^*(c_j)) + p_j(\xi_j^*(c_j),\zeta_i(m_i,c_i))\big)\,\pi(m_i,m_j|\,\rho_i(c_i),\rho_j(\widetilde{c}_j))\right]}_{=1}. \end{aligned} \tag{E.25}$$

Inequality (E.25) is, however, an equality. Hence, given our assumption that all types have a positive probability, inequality (E.23) is an equality for any $i \in \{1,2\}$, $c_i \in C_i$, and $r_j \in R_j$. This means that the randomized strategy for player i, in which each type c_i chooses the bid $\zeta_i(m_i,c_i)$ with probability $\sum_{m_j}\pi(m_i,m_j|\,\rho_i(c_i),r_j)$, is a best response to ξ_j^*, for any $r_j \in R_j$. As noted before, however, the best response in a probabilistic contest is a pure strategy and unique. Hence, $\zeta_i(m_i,c_i) = \xi_i^*(c_i)$ for all $i \in \{1,2\}$, $c_i \in C_i$, and any $m_i \in M_i$ arising with positive probability. Thus, in any communication equilibrium, all recommendations are ignored. □

Notwithstanding Theorem 3, costless unverifiable messages may indeed carry information about types. This, however, is an artefact of the hump-shape of the best-response mapping, as the following example illustrates.

Example E.4 (Irrelevant information) Suppose that $c_1^\# = 1$, $\underline{c}_2 = \frac{1}{4}$, and $\overline{c}_2 = 4$. Then, in an unbiased lottery contest,

$$x_1^\# = \left(\frac{E[\sqrt{c_2}]}{c_1 + E[c_2]}\right)^2 = \left(\frac{2q + \frac{1}{2}(1-q)}{1 + 4q + \frac{1}{4}(1-q)}\right)^2 = \frac{4}{25}, \tag{E.26}$$

regardless of posterior beliefs. Still, in equilibrium, the receiver would not make use of that information.

F. Material omitted from Section 7

This section elaborates on the welfare implications of communication in contests. We start by presenting the example of the "disclosure trap." Then, we discuss expense maximization. Finally, we deal with the problem of information design.

F.1 The "disclosure trap"

To discuss efficiency, we will compare the equilibrium scenario of full revelation (FR) with the hypothetical benchmark of mandatory concealment (MC). Let $\mathbf{C}^{\mathrm{FR}} = E[c_1 x_1^{\circ}(c_1, c_2) + c_2 x_2^{\circ}(c_1, c_2)]$ and $\mathbf{C}^{\mathrm{MC}} = E[c_1 \xi_1^*(c_1) + c_2 \xi_2^*(c_2)]$, respectively, denote total expected costs under full revelation and under mandatory concealment.[15] Further, for $i \in \{1,2\}$, let $p_i^{\mathrm{FR}} = E\left[p_i(x_1^{\circ}(c_1, c_2), x_2^{\circ}(c_1, c_2))\right]$ and $p_i^{\mathrm{MC}} = E\left[p_i(\xi_1^*(c_1), \xi_2^*(c_2))\right]$ denote player i's ex-ante probability of winning under full revelation and under mandatory concealment. Finally, likewise for $i \in \{1,2\}$, let $\Pi_i^{\mathrm{FR}} = p_i^{\mathrm{FR}} - E[c_i x_i^{\circ}(c_1, c_2)]$ and $\Pi_i^{\mathrm{MC}} = p_i^{\mathrm{MC}} - E_{c_i}[c_i \xi_i^*(c_i)]$ denote player i's ex-ante expected payoff under full revelation and mandatory concealment.

The following example illustrates the possibility that full revelation may actually be ex-ante undesirable for both contestants.

Example F.1 (**"Disclosure trap"**) The setting specified in Table F.1 satisfies Assumption 1. It can be seen that the unraveling leads the contestants into a strictly Pareto inferior outcome.

Player 1		Player 2	
$c_1^1 = 1$ $q_1(c_1^1) = 1$		$c_2^1 = 2$ $q_2(c_2^1) = 0.5$	$c_2^2 = 3$ $q_2(c_2^2) = 0.5$
$x_1^{\#} = 0.2020$		$\xi_2^{\#}(c_2^1) = 0.1158$	$\xi_2^{\#}(c_2^2) = 0.0575$
$x_1^{\circ} = 0.2222$		$x_2^{\circ} = 0.1111$	–
$x_1^{\circ} = 0.1875$		–	$x_2^{\circ} = 0.0625$
$\mathbf{C}^{\mathrm{FR}} = 0.4097$	>	$\mathbf{C}^{\mathrm{MC}} = 0.4040$	
$p_1^{\mathrm{FR}} = 0.7083 > p_1^{\mathrm{MC}} = 0.7071$		$p_2^{\mathrm{FR}} = 0.2917 < p_2^{\mathrm{MC}} = 0.2929$	
$\Pi_1^{\mathrm{FR}} = 0.5035 < \Pi_1^{\mathrm{MC}} = 0.5050$		$\Pi_2^{\mathrm{FR}} = 0.0868 < \Pi_2^{\mathrm{MC}} = 0.0909$	

Table F.1 Equilibrium bids under full revelation and mandatory concealment.

The example illustrates that the option to disclose verifiable information may be undesirable for a contestant. The reason is an externality that the self-disclosing marginal type imposes on the silent submarginal types. The externality is a virtual one only, because two type realizations of the same contestant never coexist. Notwithstanding, the inability to commit leads to a situation in which the privately informed player loses in expected terms by the unraveling.

F.2 Expense maximization

The following result shows that, even though full revelation need not be in the interest of an informed contestant, a contest organizer maximizing total expected expenses may well find that outcome preferable to full concealment.

Proposition F.1 (Expense maximization) *Consider a uniformly asymmetric, unbiased lottery contest. Suppose that the type of the favorite is public information, whereas the underdog has at least two possible type realizations. Assume also that, under mandatory concealment, all types are active. Then,*

(i) $\mathbf{C}^{\mathrm{FR}} > \mathbf{C}^{\mathrm{MC}}$ *(in both cases, expected costs split evenly between the players);*[16]

(ii) the underdog's (the favorite's) ex-ante probability of winning is strictly lower (strictly higher) under full revelation than under mandatory concealment, i.e., $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$ $(p_1^{\mathrm{FR}} > p_1^{\mathrm{MC}})$*; and*

(iii) the ex-ante payoff for the underdog is strictly lower under full revelation than under mandatory concealment, i.e., $\Pi_2^{\mathrm{FR}} < \Pi_2^{\mathrm{MC}}$.[17]

[15] $E[.] = E_{c_1,c_2}[.]$ denotes the ex-ante expectation.

[16] Thus, the effort of the favorite is strictly higher under full revelation than under mandatory concealment. The expected effort of the underdog, however, may either rise or fall, depending on parameters.

[17] The payoff comparison for the favorite is ambiguous, i.e., depending on parameters, it may be that $\Pi_1^{\mathrm{FR}} \geq \Pi_1^{\mathrm{MC}}$, or as in Example F.1, that $\Pi_1^{\mathrm{FR}} < \Pi_1^{\mathrm{MC}}$.

Proof. (i) Let $c_1^{\#} \in C_1$ denote the public type of the favorite. For the unbiased lottery contest, an interior equilibrium may be easily derived from the corresponding first-order conditions (Hurley and Shogren, 1998a; Epstein and Mealem, 2013; Zhang and Zhou, 2016). In our set-up, this yields equilibrium bids $x_1^{\#} = E[\sqrt{c_2}]^2/(c_1^{\#} + E\left[c_2\right])^2$ for player 1, and $\xi_2^{\#}(c_2) = \sqrt{x_1^{\#}/c_2} - x_1^{\#}$ for any $c_2 \in C_2$, where we dropped the subscript c_2 from the expectation operator. Using these expressions, total expected costs under mandatory concealment are easily derived as $\mathbf{C}^{\mathrm{MC}} = c_1^{\#} x_1^{\#} + E[c_2 \xi_2^{\#}(c_2)] = 2c_1^{\#} E[\sqrt{c_2}]^2/(c_1^{\#} + E[c_2])^2$. Note that this formula entails, in particular, the complete-information case where c_2 is public as well. Therefore, being an expectation over such complete-information scenarios, total expected costs under full revelation amount to $\mathbf{C}^{\mathrm{FR}} = 2E[c_1^{\#} c_2/(c_1^{\#} + c_2)^2]$. To compare the two expressions, we apply Lemma G.1 with $Y = \sqrt{c_2/c_1^{\#}}$ and $g(x,y) = g_1(x,y) \equiv 2x^2/\left(1+y\right)^2$. It suffices to show that, for any $x > 1$, $y \geq x^2$, $d_x > 0$, $d_y > 0$ such that $\frac{d_y}{d_x} > \frac{y-1}{x-1}$, the quadratic form

$$(d_x \; d_y)\left(H_{g_1}(x,y)\right)\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{4{d_x}^2}{\left(1+y\right)^2}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)\left(1 - \frac{3x}{1+y}\frac{d_y}{d_x}\right) \tag{F.1}$$

attains a positive value. To see this, one checks that

$$\frac{x}{y+1} \cdot \frac{d_y}{d_x} > \underbrace{\frac{x}{y+1} \cdot \frac{y-1}{x-1}}_{\text{increasing in } y} \geq \frac{x}{x^2+1} \cdot \frac{x^2-1}{x-1} = \frac{x^2+x}{x^2+1} > 1. \tag{F.2}$$

Clearly then, the right-hand side of (F.1) is positive. This proves the claim. It follows that

$$\mathbf{C}^{\mathrm{FR}} = E\left[\frac{2(c_2/c_1^{\#})}{(1+(c_2/c_1^{\#}))^2}\right] > \frac{2E\left[\sqrt{c_2/c_1^{\#}}\right]^2}{(1+E[c_2/c_1^{\#}])^2} = \mathbf{C}^{\mathrm{MC}}, \tag{F.3}$$

i.e., total expected costs are indeed strictly higher under full revelation than under mandatory concealment. In particular, given that, by (B.15), expected costs in the lottery contest are the same across contestants, and given that the favorite's type is public, the favorite exerts a higher effort under full revelation than under mandatory concealment. (ii) From the explicit expressions for the equilibrium bids given above, player 1's probability of winning is easily determined as $p_1^{\mathrm{MC}} = E[\sqrt{c_2}]^2/(c_1^{\#} + E\left[c_2\right])$ under mandatory concealment, and by $p_1^{\mathrm{FR}} = E[c_2/(c_1^{\#} + c_2)]$ under full revelation. Again, we apply Lemma G.1 for $Y = \sqrt{c_2/c_1^{\#}}$, using this time the mapping $g(x,y) = g_2(x,y) \equiv x^2/(1+y)$. Suppose that $x > 1$, $y \geq x^2$, $d_x > 0$, and $d_y > 0$. Then, clearly,

$$(d_x \; d_y)\left(H_{g_2}(x,y)\right)\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \frac{2{d_x}^2}{1+y}\left(1 - \frac{x}{1+y}\frac{d_y}{d_x}\right)^2 \geq 0. \tag{F.4}$$

Moreover, from relationship (F.2), inequality (F.4) is even strict, which implies strict convexity of g_2 along the relevant linear path segment. Thus, we have

$$p_1^{\mathrm{FR}} = E\left[\frac{(c_2/c_1^{\#})}{1+(c_2/c_1^{\#})}\right] > \frac{E[\sqrt{c_2/c_1^{\#}}]^2}{1+E[c_2/c_1^{\#}]} = p_1^{\mathrm{MC}}, \tag{F.5}$$

and, consequently, also $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$. (iii) Since expected costs are equal across players in the lottery contest, ex-ante expected payoffs for the underdog are given by $\Pi_2^{\mathrm{FR}} = p_2^{\mathrm{FR}} - \frac{\mathbf{C}^{\mathrm{FR}}}{2}$ under full revelation, and by $\Pi_2^{\mathrm{MC}} = p_2^{\mathrm{MC}} - \frac{\mathbf{C}^{\mathrm{MC}}}{2}$ under mandatory concealment. As seen above, $p_2^{\mathrm{FR}} < p_2^{\mathrm{MC}}$ and $C^{\mathrm{FR}} > C^{\mathrm{MC}}$. Hence, $\Pi_2^{\mathrm{FR}} < \Pi_2^{\mathrm{MC}}$, as claimed. □

F.3 Information design

Next, we assume that an informed contest designer chooses a signal to maximize some policy objective (Wasser, 2013a; Denter et al., 2014; Zheng and Zhou, 2016). Upon receiving the realization of the signal, the uninformed player updates her belief and the contest takes place. The following result characterizes the optimal signal for three specific policy objectives, viz. maximizing total expected efforts, maximizing total expected payoffs, and minimizing the expected quadratic distance of players' winning probabilities.[18]

Proposition F.2 (Information design) *Consider an unbiased lottery contest where player 1's type $c_1^{\#}$ is public, while player 2's type $c_2 \in \{\underline{c}_2, \overline{c}_2\}$ is private. Suppose also that the interiority assumption of Lemma E.1 holds. Then, for a contest designer:*

(i) maximizing total expected efforts, full disclosure (full concealment, any signal) is optimal if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$ (if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$, if $c_1^{\#} = \sqrt{\underline{c}_2\overline{c}_2}$);

(ii) maximizing total expected payoffs, it is optimal to delegate the problem to player 2, i.e., to use the signal characterized in Proposition E.1;

(iii) minimizing $E_{c_2}[(p_1 - p_2)^2]$, it is optimal to use the signal characterized in part (i).

Proof. (i) This result follows immediately from Zhang and Zhou (2016, Prop. 3) by replacing valuations by reciprocals of marginal costs. (ii) In an unbiased lottery contest with incomplete information about marginal costs, the ex-ante expected expenses are identical across players. Moreover, the prize is always assigned to one player. Therefore, maximizing total expected payoffs is equivalent to minimizing player 1's expenses, $c_1^{\#}x_1^{\#}$. However, as shown in the proof of Lemma D.1, all types $c_2 \in C_2$ prefer a strictly lower bid $x_1^{\#}$ over any higher bid. The claim follows. (iii) In an unbiased lottery contest, we have

$$\frac{1}{4}\left\{1 - E_{c_2}\left[\left(p_1(x_1^{\#}, \xi_2^{\#}(c_2)) - p_2(x_1^{\#}, \xi_2^{\#}(c_2))\right)^2\right]\right\} = E_{c_2}\left[p_1(x_1^{\#}, \xi_2^{\#}(c_2))(1 - p_1(x_1^{\#}, \xi_2^{\#}(c_2)))\right] = x_1^{\#}c_1^{\#}. \quad \text{(F.6)}$$

Hence, minimizing the expected quadratic distance between players' winning probabilities is equivalent to maximizing player 1's expenses. But, by the arguments just explained, this is equivalent to the problem considered under part (i). The proposition follows. □

Part (i) says that, to maximize expected efforts, full disclosure is optimal if player 1 is comparably strong (i.e., if $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$), while full concealment is optimal if player 1 is comparably weak (i.e., if $c_1^{\#} > \sqrt{\underline{c}_2\overline{c}_2}$), with any signal being optimal in the knife-edge case where $c_1^{\#} = \sqrt{\underline{c}_2\overline{c}_2}$. Part (i) is a straightforward reformulation of a well-known result due to Zhang and Zhou (2016).[19] For parts (ii) and (iii), however, we have not found a suitable reference. Part (ii) is a statement about decentralization. Part (iii) may not be too surprising. Indeed, under the assumptions made, minimizing the expected quadratic distance turns out to be equivalent to maximizing total expected efforts.[20]

An interesting policy objective is also the maximization of the expected highest bid. In general, that problem may be difficult. Imposing Assumption 1, however, the problem simplifies. Indeed, since player 1 is known to submit the highest bid, and ex-ante expected costs are identical for both bidders, the problem becomes equivalent to the one considered in part (i) above.

[18] Still another policy objective, the maximization of the expected highest bid, will be considered below.

[19] In their case, however, private information is about valuations. Zhang and Zhou (2016) also offer an algorithm for solving the case with $K_2 \geq 3$ types. With more than two types, if the uninformed player is strong enough, full disclosure is optimal, otherwise pooling the highest two valuations together and fully separating the others maximizes total efforts. The paper points out the difficulties that arise in a setting with two-sided incomplete information, namely (i) the multi-dimensional state of nature of both contestants' valuations which complicates the persuasion stage, (ii) the private information on two sides, where the simplifying step of the analysis of Kamenica and Gentzkow (2011) cannot be applied, and (iii) the equilibrium characterization which is in general not available. More recently, some progress on this problem has been made by Serena (2022).

[20] There is an intuitive tension between part (iii) and the discussion of expense maximization. Specifically, in a setting with a comparably strong player 1 in which both Assumption 1 holds and $c_1^{\#} < \sqrt{\underline{c}_2\overline{c}_2}$, we find here that the optimal signal entails full disclosure, whereas Proposition F.1(ii) implies mandatory concealment. To understand what is going on, note that Proposition F.2(iii) works with a quadratic distance of probabilities, whereas Proposition F.1(ii) works with ex-ante winning probabilities. Therefore, the policy objective considered here, intuitively speaking, places overproportional weight on the most lopsided encounters, whereas the earlier discussion weights all encounters according to their ex-ante probability of occurrence, which explains the difference in conclusions.

G. Refinement of Jensen's inequality

Some of our examples fall into the tractable class of lottery contests with one-sided incomplete information (Hurley and Shogren, 1998a; Zhang and Zhou, 2016). Below, we derive a variant of Jensen's inequality that allows to prove certain payoff inequalities that cannot be easily obtained otherwise. These payoff inequalities relate to sequentially taken disclosure decisions, the option to shut down communication, and the maximization of expenses. We will state conditions that are sufficient to derive inequalities of the type $E\left[g\left(Y,Y^2\right)\right] > g\left(E[Y],E[Y^2]\right)$ for a function g in two arguments and a nondegenerate random variable $Y > 0$ with finite support. As can be seen, the inequality makes use of the second moment of Y, which explains why it can be sharper than Jensen's inequality. The inequality is strict as a result of our assumption that Y is not degenerate.[21]

Assuming that g is twice continuously differentiable, and given $x > 0$, $y > 0$, $d_x > 0$, $d_y > 0$, we will say that g is *directionally strictly convex* at (x,y) along (d_x,d_y) if $(d_x\ d_y)\left(H_g(x,y)\right)(d_x\ d_y)^T > 0$, where $H_g(x,y)$ denotes the Hessian of g, and T denotes transposition.

Lemma G.1 (Jensen's inequality refined) *Suppose that one of the following two conditions holds:*
(i) $Y > 1$ with probability one, and g is directionally strictly convex at (x,y) along (d_x,d_y) whenever $y \geq x^2 > 1$ and $d_y/d_x > (y-1)/(x-1)$.
(ii) $Y \in (0,1)$ with probability one, and g is directionally strictly convex at (x,y) along (d_x,d_y) whenever $1 > y \geq x^2$ and $d_y/d_x < (1-y)/(1-x)$.
Then, $E\left[g\left(Y,Y^2\right)\right] > g\left(E[Y],E[Y^2]\right)$.

Proof. (i) By induction. Assume first that Y has precisely two possible realizations $y_1, y_2 \in (1,\infty)$. Without loss of generality, $y_1 < y_2$. Consider the auxiliary mapping $f : [0,1] \to R^2$ defined through

$$f(t) = (1-t)\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t\begin{pmatrix} y_2 \\ y_2^2 \end{pmatrix} \qquad (t \in [0,1]), \tag{G.1}$$

as illustrated in Figure G.1(a). By assumption, g is strictly convex along the straight line described by f.[22] In particular, the composed mapping $g \circ f$ is strictly convex. Therefore, if t is considered a random variable that assumes the value $t = 0$ with probability $q_1 = \mathrm{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $q_2 = 1 - q_1 = \mathrm{pr}(Y = y_2) > 0$, then

$$\begin{aligned} E[g(Y,Y^2)] &= E[g(f(t))] && \text{(G.2)} \\ &> g(f(E[t])) && \text{(G.3)} \\ &= g\left(q_1 y_1 + (1-q_1)y_2, q_1 y_1^2 + (1-q_1)y_2^2\right) && \text{(G.4)} \\ &= g([E[Y],E[Y^2]). && \text{(G.5)} \end{aligned}$$

This proves the claim if Y has two realizations. Suppose that the claim has been shown for $K \geq 2$ realizations, and assume that Y has $K+1$ realizations $y_1 < \ldots < y_{K+1}$, with respective probabilities $q_k = \mathrm{pr}(Y = y_k) > 0$, where $k = 1,\ldots,K+1$. Consider the random variable Y' that attains value y_k, for $k = 2,\ldots,K+1$, with probability

$$q_k' = \frac{q_k}{1-q_1} = \frac{q_k}{\sum_{\kappa=2}^{K+1} q_\kappa}. \tag{G.6}$$

Thus, Y' follows a conditional distribution after learning $Y \neq y_1$. In particular, $E[Y] = q_1 y_1 + (1-q_1)E[Y']$ and $E[Y^2] = q_1 y_1^2 + (1-q_1)E[(Y')^2]$. Moreover, by the induction hypothesis, $E\left[g\left(Y',(Y')^2\right)\right] > g\left(E[Y'],E[(Y')^2]\right)$. As above, we define an auxiliary mapping

$$\widetilde{f}(t) = (1-t)\begin{pmatrix} y_1 \\ y_1^2 \end{pmatrix} + t\begin{pmatrix} E[Y'] \\ E[(Y')^2] \end{pmatrix} \qquad (t \in [0,1]). \tag{G.7}$$

[21] For alternative extensions of Jensen's inequality, see Pittenger (1990), Guljaš et al. (1998), and Liao and Berg (2017). However, those results do not deliver the payoff comparisons mentioned above.

[22] To see this, let $x = (1-t)y_1 + ty_2 > 1$, $y = (1-t)y_1^2 + ty_2^2 \geq x^2$, $d_x = y_2 - y_1 > 0$, and $d_y = y_2^2 - y_1^2 > 0$. Then, $d_y/d_x = y_2 + y_1 > y_2 + 1 \geq (y-1)/(x-1)$, so that the precondition in (i) indeed holds true.

Clearly, $E[(Y')^2] > E[Y']^2$. Therefore, as illustrated in Figure G.1(b), the vector that directs from $(y_1, (y_1)^2)$ to $(E[Y'], E[(Y')^2])$ is steeper than the vector that directs from $(y_1, (y_1)^2)$ to $(E[Y'], E[Y']^2)$. Hence, g is strictly convex also along the linear path described by $\widetilde{f}$.[23] Thus, $g \circ \widetilde{f}$ is strictly convex.

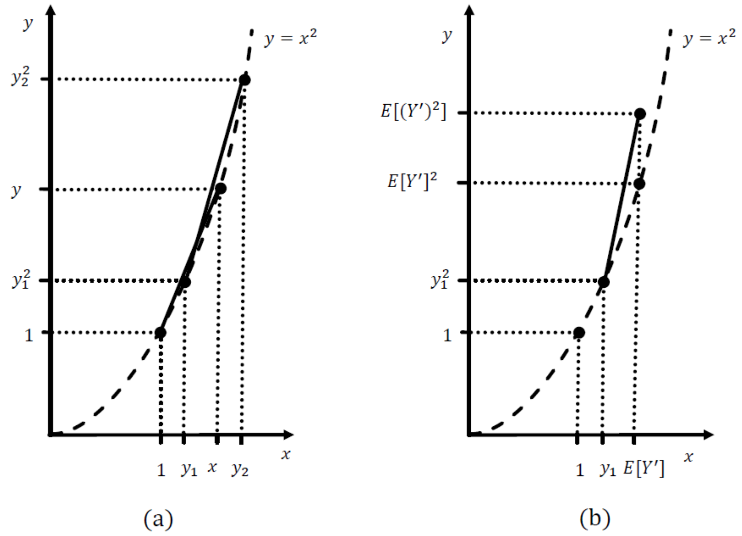


Figure G.1 A refinement of Jensen's inequality.

Therefore, considering t as a random variable that assumes the value $t = 0$ with probability $q_1 = \mathrm{pr}(Y = y_1) > 0$ and the value $t = 1$ with probability $1 - q_1 > 0$, the relationships derived above imply

$$
\begin{aligned}
E[g(Y, Y^2)] &= q_1 g(y_1, y_1^2) + (1 - q_1) E[g(Y', (Y')^2] && \text{(G.8)} \\
&> q_1 g(y_1, y_1^2) + (1 - q_1) g\left(E[Y'], E[(Y')^2]\right) && \text{(G.9)} \\
&= E[g(\widetilde{f}(t))] && \text{(G.10)} \\
&> g(\widetilde{f}(E[t])) && \text{(G.11)} \\
&= g\left(q_1 y_1 + (1 - q_1) E[Y'], q_1 y_1^2 + (1 - q_1) E[(Y')^2]\right) && \text{(G.12)} \\
&= g(E[Y], E[Y^2]). && \text{(G.13)}
\end{aligned}
$$

Thus, the claim holds for $K + 1$ realizations. This completes the induction, and thereby, the proof of the claim. (ii) The proof is entirely analogous to the one just given and therefore omitted. □

H. Additional references

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[23] Indeed, letting $x = (1-t)y_1 + tE[Y'] > 1$, $y = (1-t)y_1^2 + tE[(Y')^2] > x^2$, $d_x = E[Y'] - y_1 > 0$, and $d_y = E[(Y')^2] - y_1^2 > 0$, we see that $d_y/d_x = (E[(Y')^2] - y_1^2)/(E[Y'] - y_1) > (E[(Y')]^2 - y_1^2)/(E[Y'] - y_1) = E[Y'] + y_1 > 1 + y_1 = (y-1)/(x-1)$, so that the precondition in (i) holds true also in this case.